

dark matter

Milena Crnogorčević

Postdoctoral Fellow at the Oskar Klein Centre
→ GRAPPA, University of Amsterdam [Fall 2026]

International School of Cosmic Ray Astrophysics
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Lecture 1/3



dark matter

1

Dark Matter Introduction

2

Dark Matter Candidates and Detection
Strategies

3

Indirect Detection

dark matter

1

Dark Matter Introduction

2

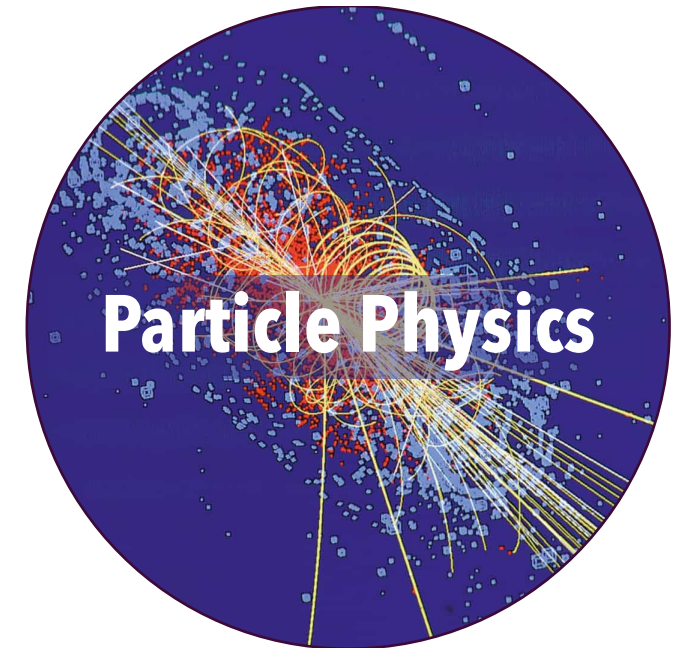
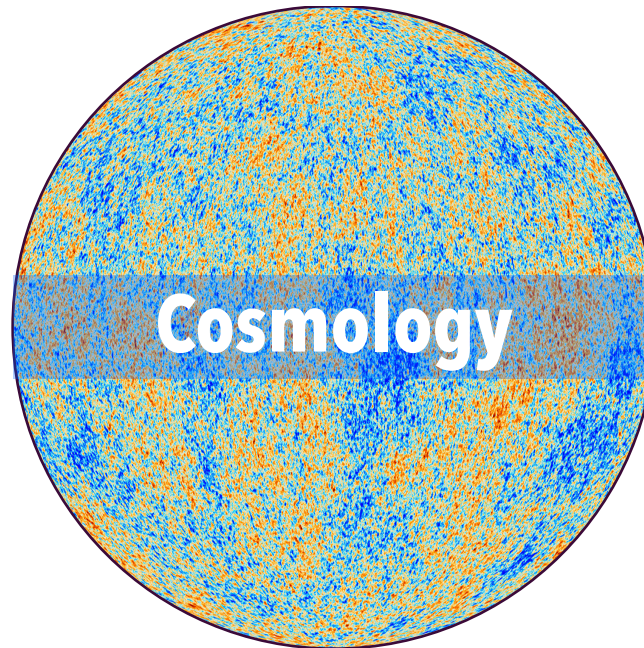
Dark Matter Candidates and Detection
Strategies

3

Indirect Detection

Why do we think dark matter exists?
What have observations taught us about it?

dark matter



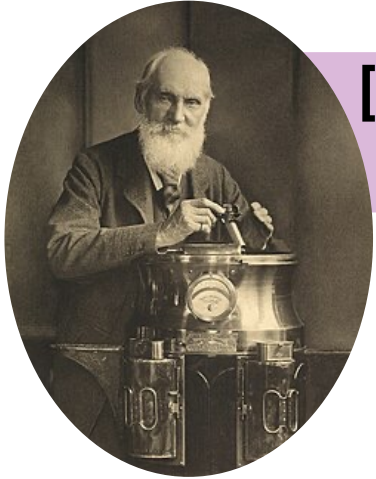
Historical context

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How do we measure mass of astronomical systems?

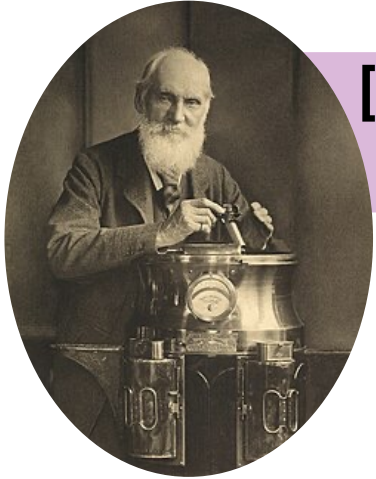


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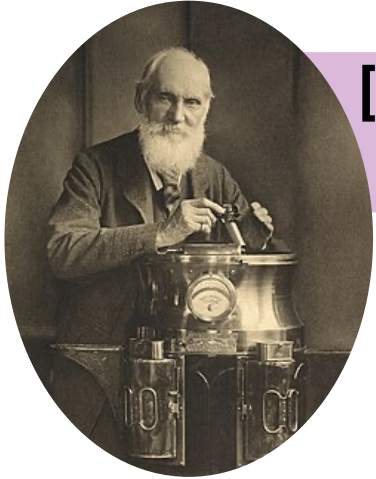


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→ galaxies & clusters: unresolved mixture of stellar masses, ages...

$$M_* = \boxed{Y_*} L$$

depends on metallicity, initial mass function, remnants, observing band, etc.

Zwicky (1937) compares the dynamically-measured Y_* of the Coma to the local $Y_* \sim 3 \frac{M_\odot}{L_\odot}$.

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For a particle k :

- $\mathbf{r}_k \cdot \mathbf{p}_k > 0$: the particle is moving outward;
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Remember, $\dot{\mathbf{r}}_k = \mathbf{v}_k$, $\mathbf{p}_k = m_k \mathbf{v}_k$, and $\dot{\mathbf{p}}_k = \mathbf{F}_k$.

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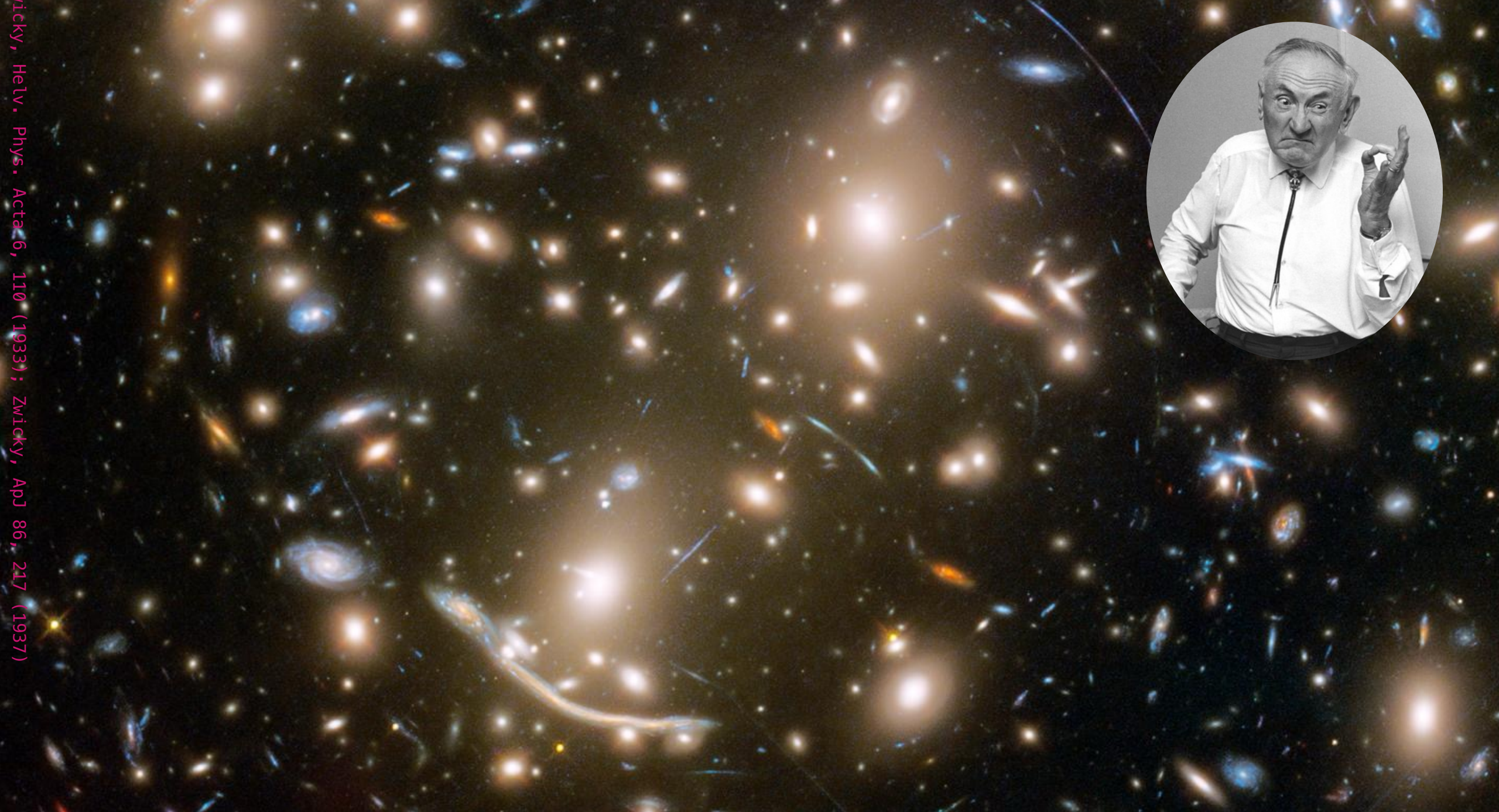
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- For conservative forces from a potential U , then $\mathbf{F}_k = -\frac{\partial U}{\partial \mathbf{r}_k}$.
- Then, for $U(r) = Ar^n \Rightarrow -\sum_{k=1}^N \mathbf{r}_k \cdot \mathbf{F}_k = nU_{\text{tot}}$.
- Finally, for gravity $n = -1$, thus:

$$\sum_{k=1}^N \mathbf{r}_k \cdot \mathbf{F}_k = U_{\text{tot}}$$

Virial theorem: recap

$$2\langle T \rangle + \langle U_{\text{tot}} \rangle = 0$$



Wick, Helv. Phys. Acta 6, 110 (1933); Zwicky, ApJ 86, 217 (1937)

For the N number of galaxies with mass m in the cluster,

$$T = N \frac{m}{2} \langle v^2 \rangle \simeq \frac{M_{\text{dyn}}}{2} \langle v^2 \rangle$$



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Applying the virial theorem,

$$M_{\text{dyn}} \simeq \mathcal{O}(1) \frac{\langle v^2 \rangle R}{G}$$

spectroscopy

distance +
angular size



The Redshift of Extragalactic Nebulae

by F. Zwicky.

(16.II.33.)



"In order to obtain the observed value of [velocity], the average density in the Coma system would have to be at least **400 times larger** than that derived on the grounds of observations of luminous matter.

If this would be confirmed **we would get the surprising result that dark matter is present in much greater amount than luminous matter.**"

Note: Zwicky (1933) assumes 800 galaxies with an average mass of $10^9 M_{\odot}$, and compares it to the dynamical mass. Zwicky (1937) reaches a similar conclusion, with $Y_* \sim 530$ compared to the local value of ~ 3 .

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Does the virial theorem apply to the Coma cluster?

✓-ish



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Was the luminous mass right?

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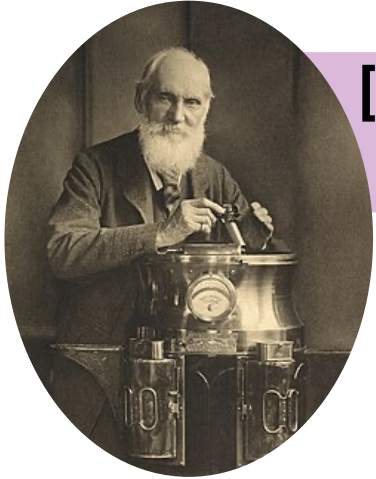
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Was the distance scale correct?

✗

No. $M/L \sim h$ ---Zwicky's ratio inflated ~ 8 times.

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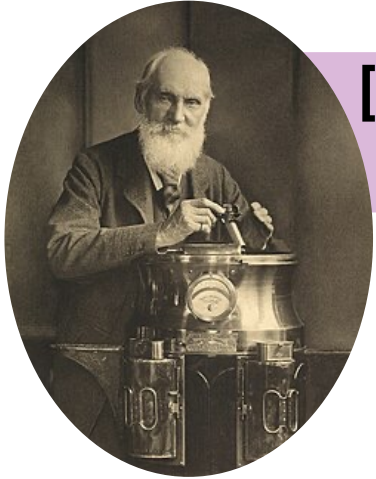


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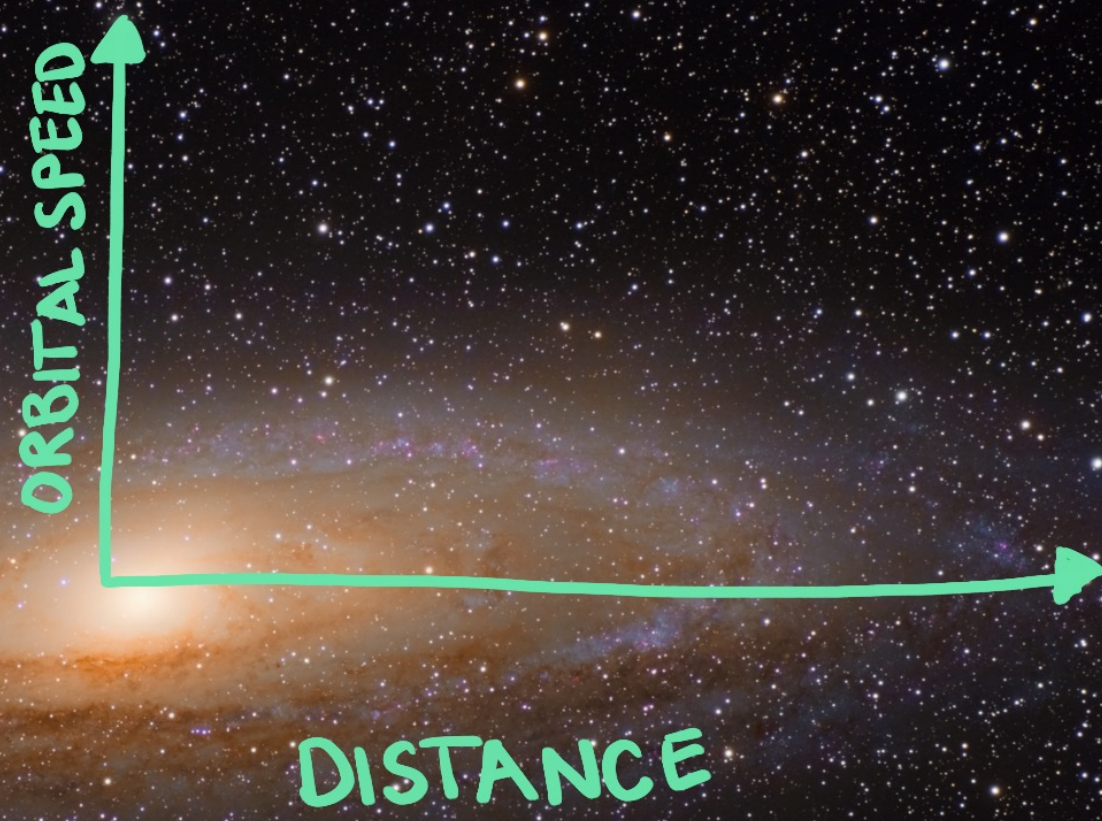
[1970s and '80s] A convergence of evidence – Radio and optical rotation curves, dynamical arguments, and halo models establish that galaxy masses extend far beyond their visible disks.



G. Bertone and D. Hooper, “History of Dark Matter,”
Reviews of Modern Physics **90**, 045002 (2018), [arXiv:1605.04909](https://arxiv.org/abs/1605.04909).

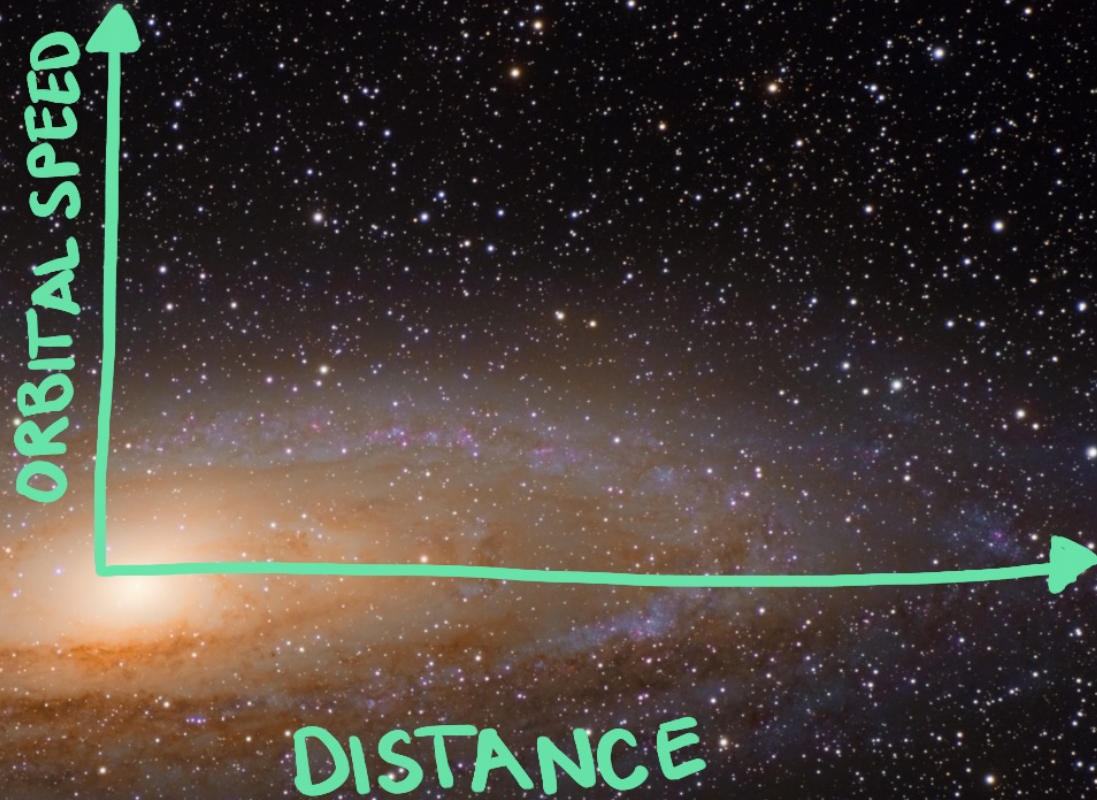
[optical] Rubin, Ford, & Thonnard; [radio] Roberts, Rots, Rogstad, Shostak, Bosma
[interpretation] Freeman · Einasto, Kaasik & Saar · Ostriker, Peebles & Yahil





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$$M(< R) \equiv 4\pi \int_0^R r^2 \rho(r) dr$$

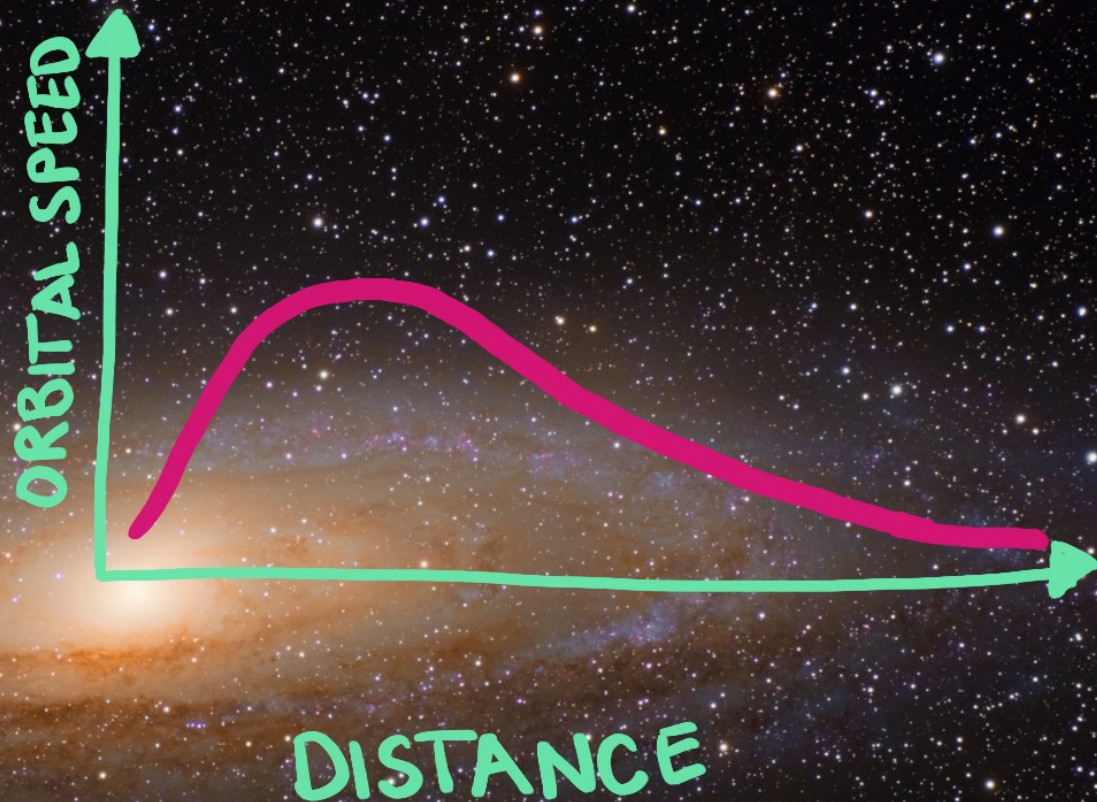


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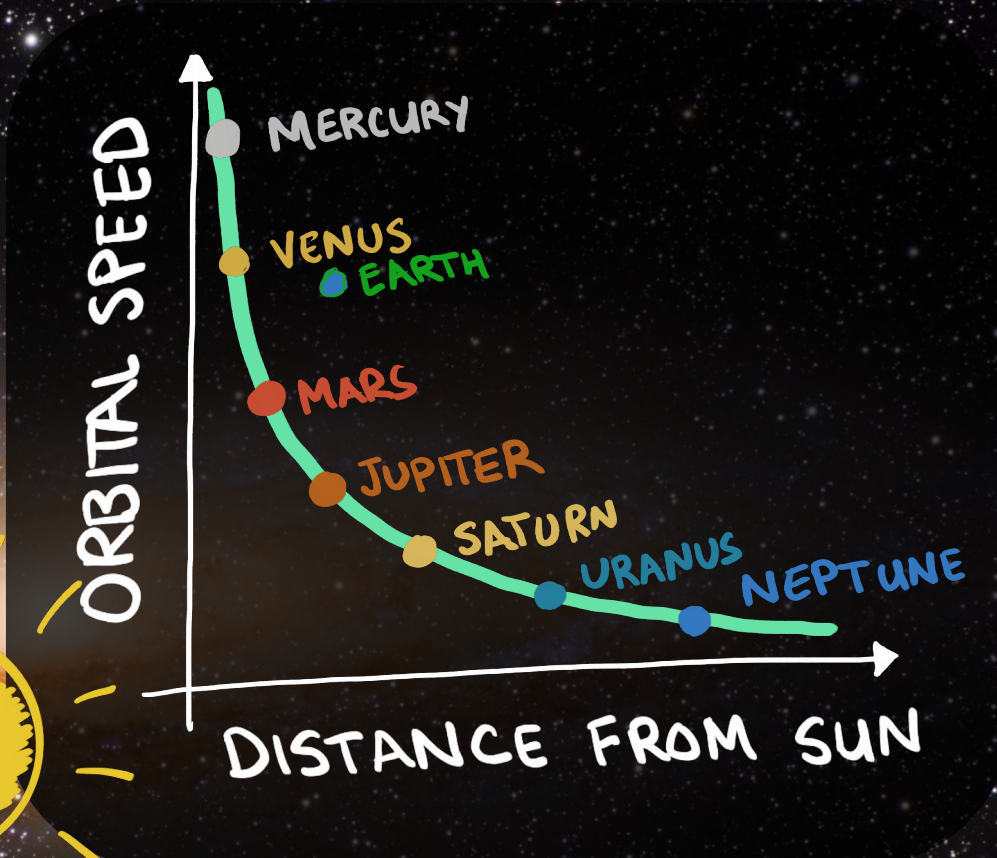


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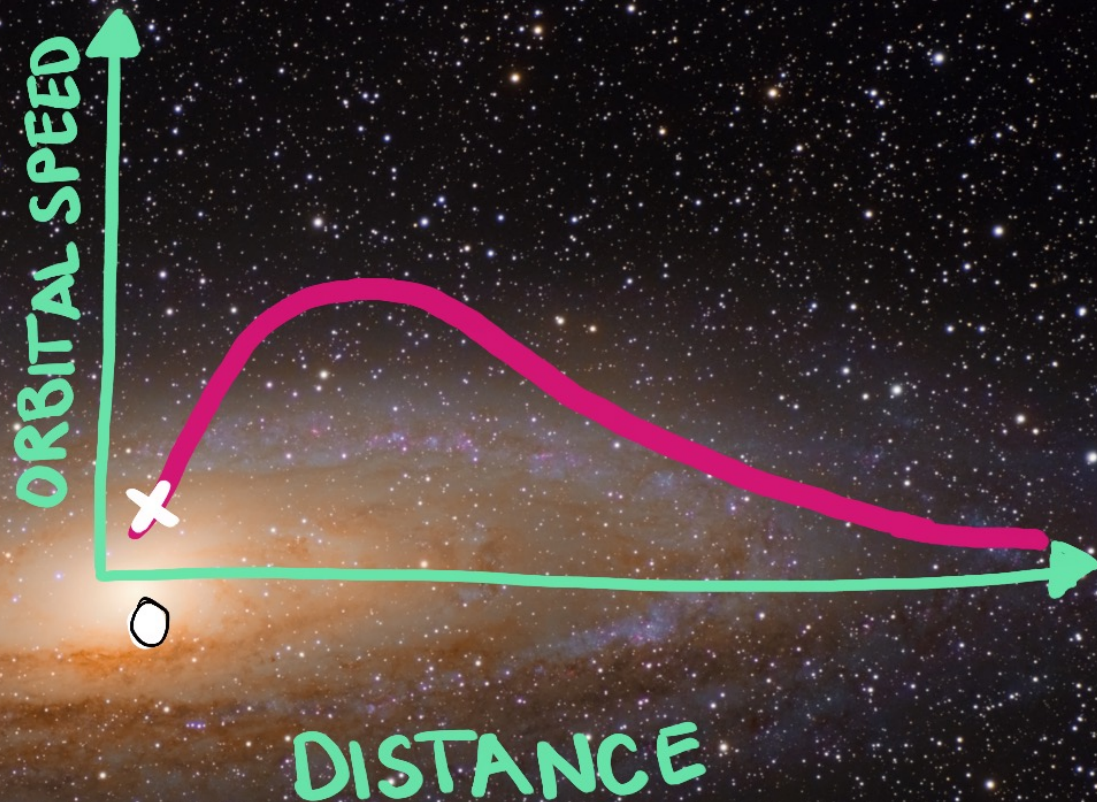


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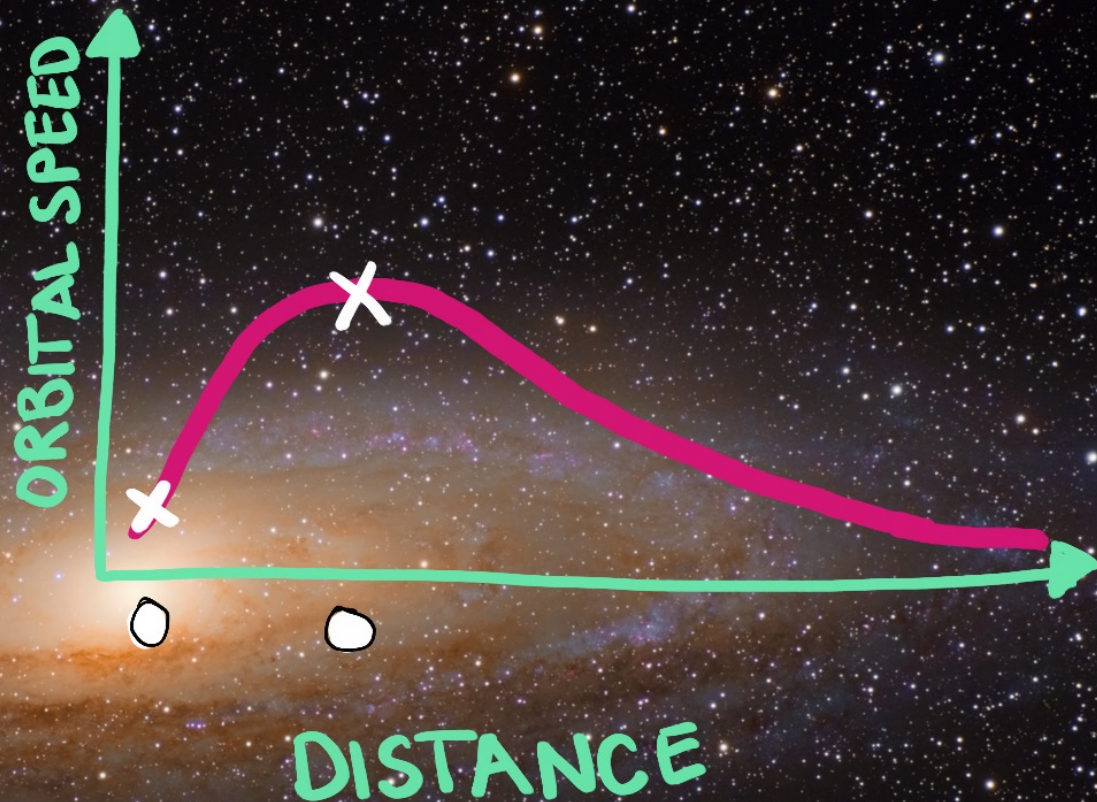


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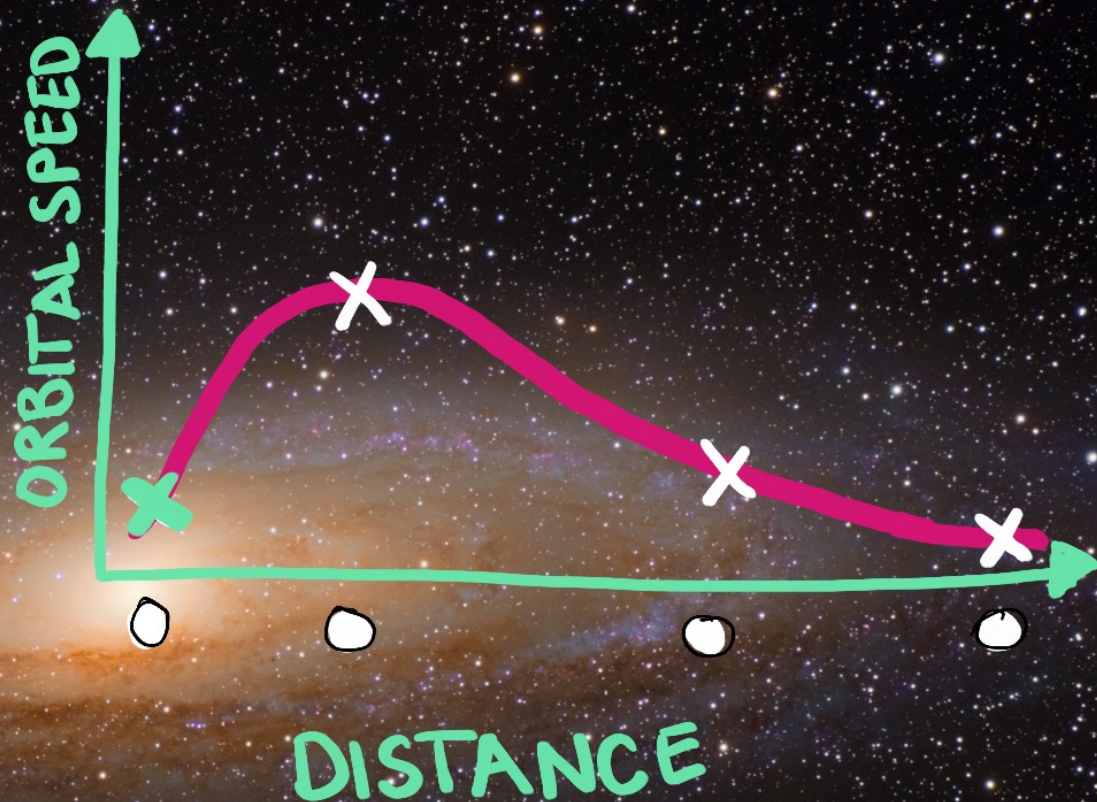


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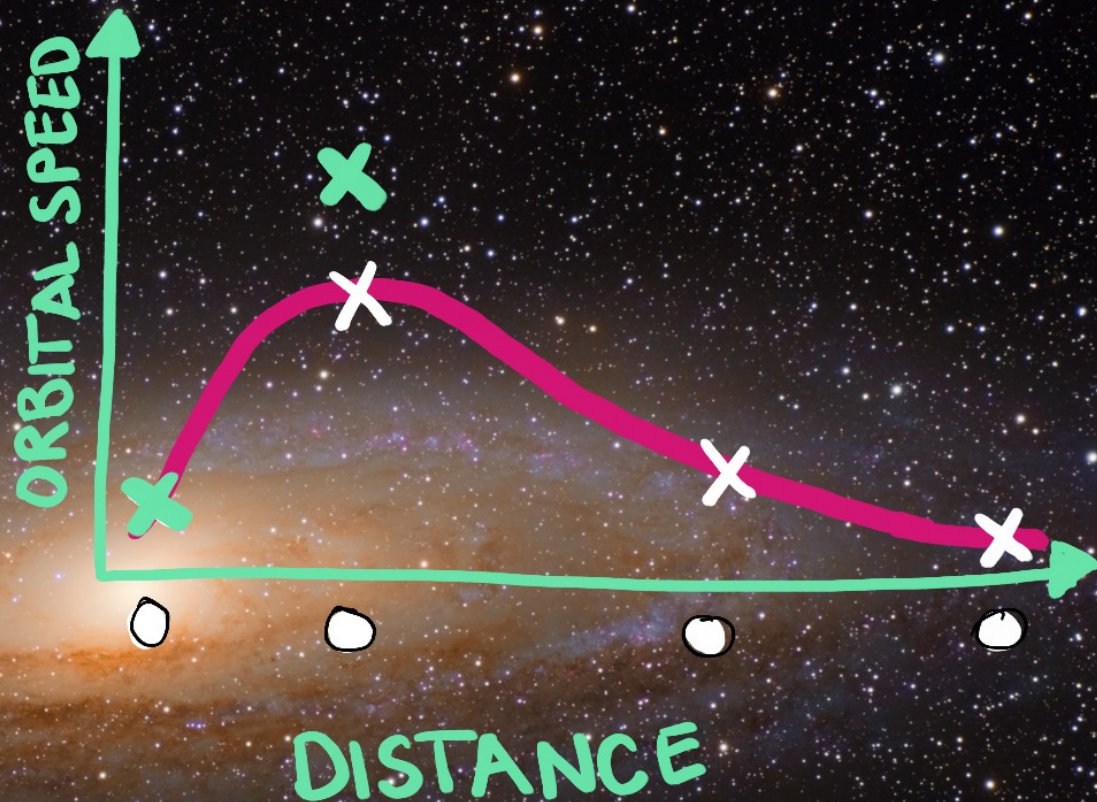


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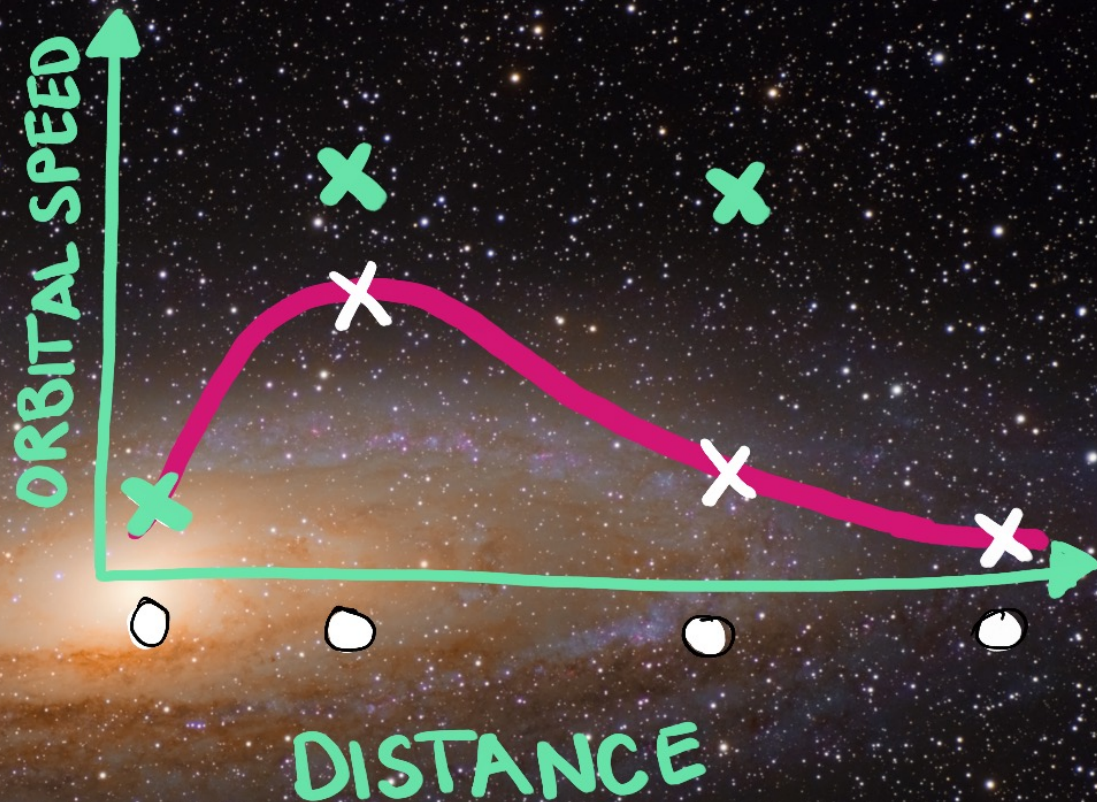


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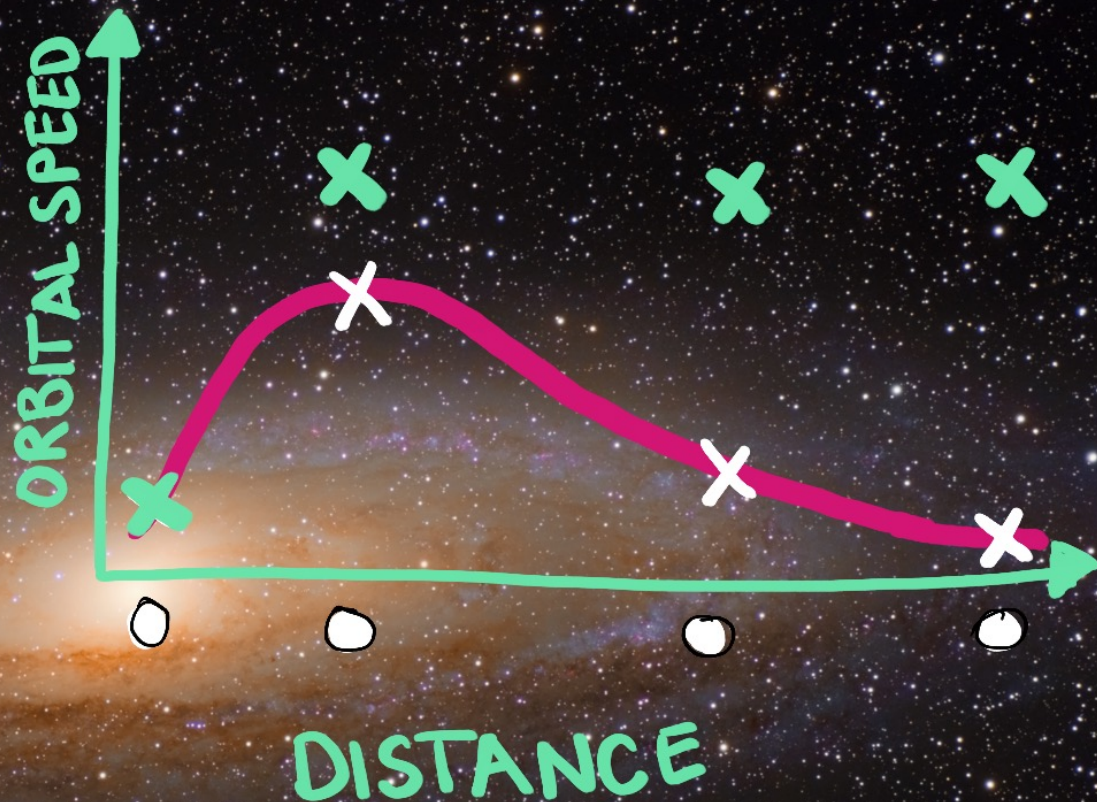


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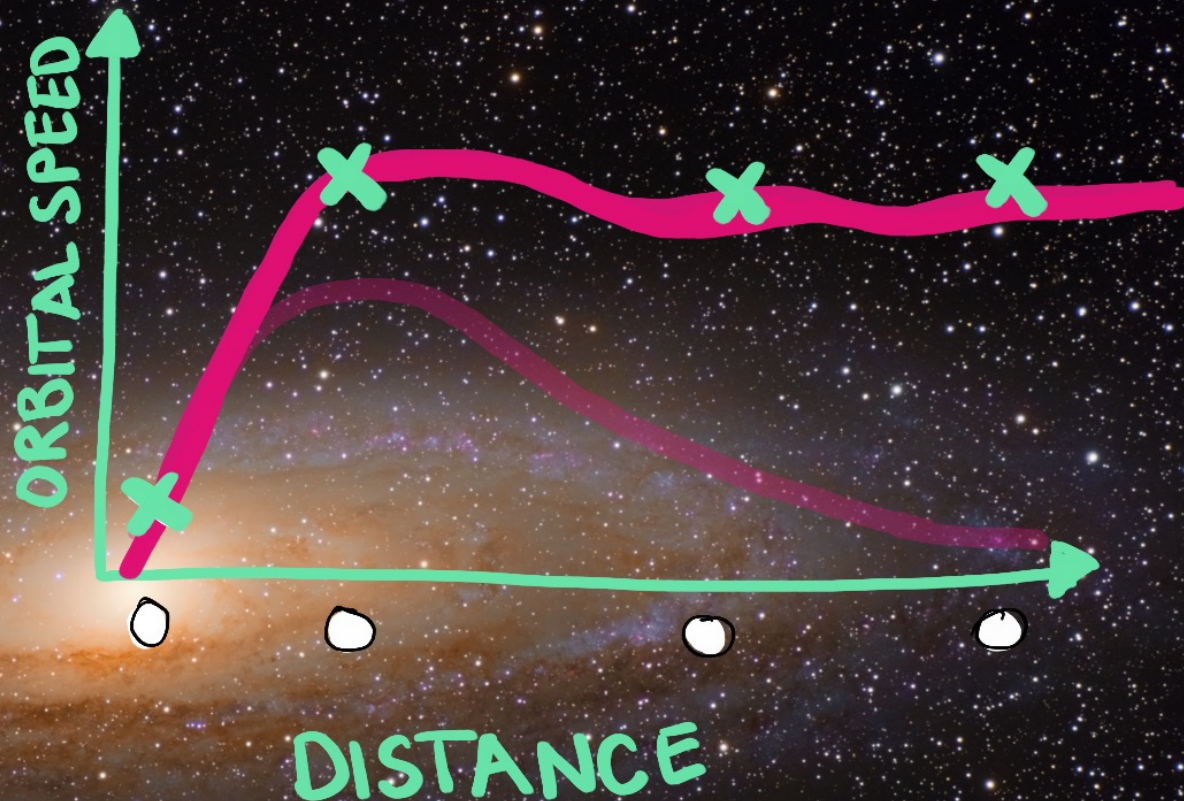
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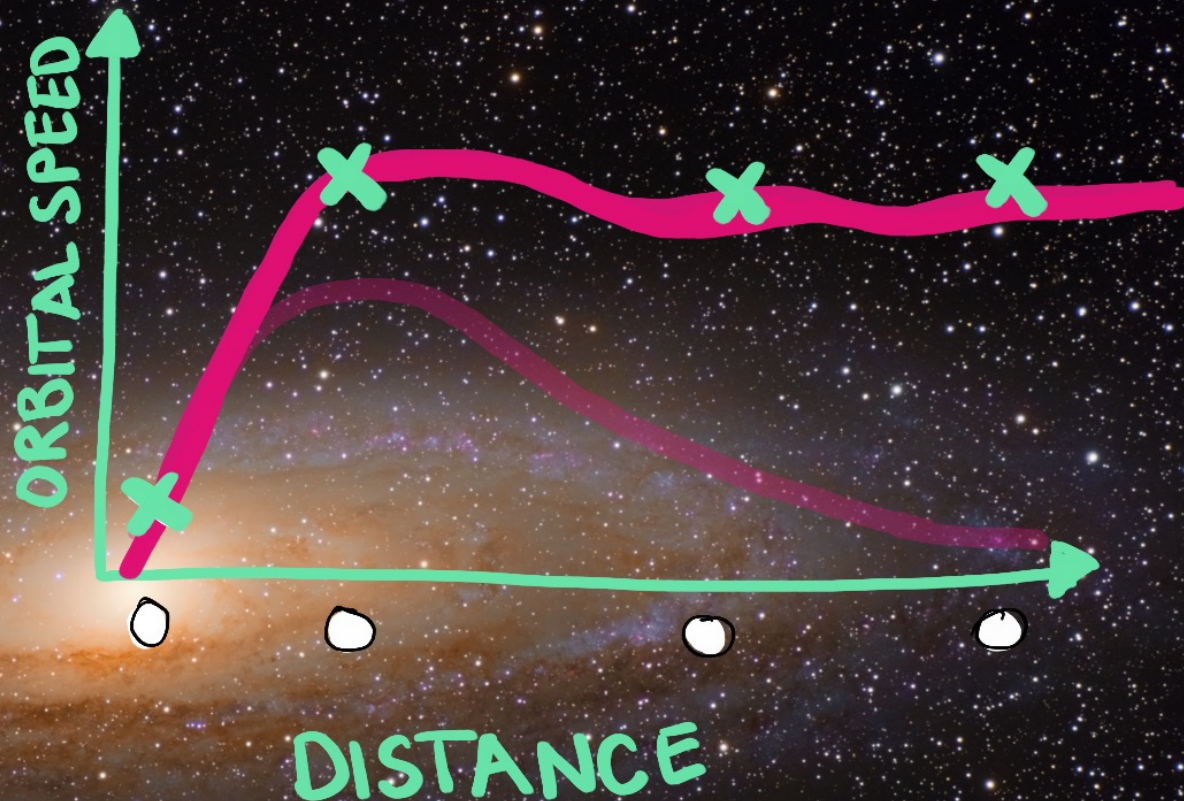
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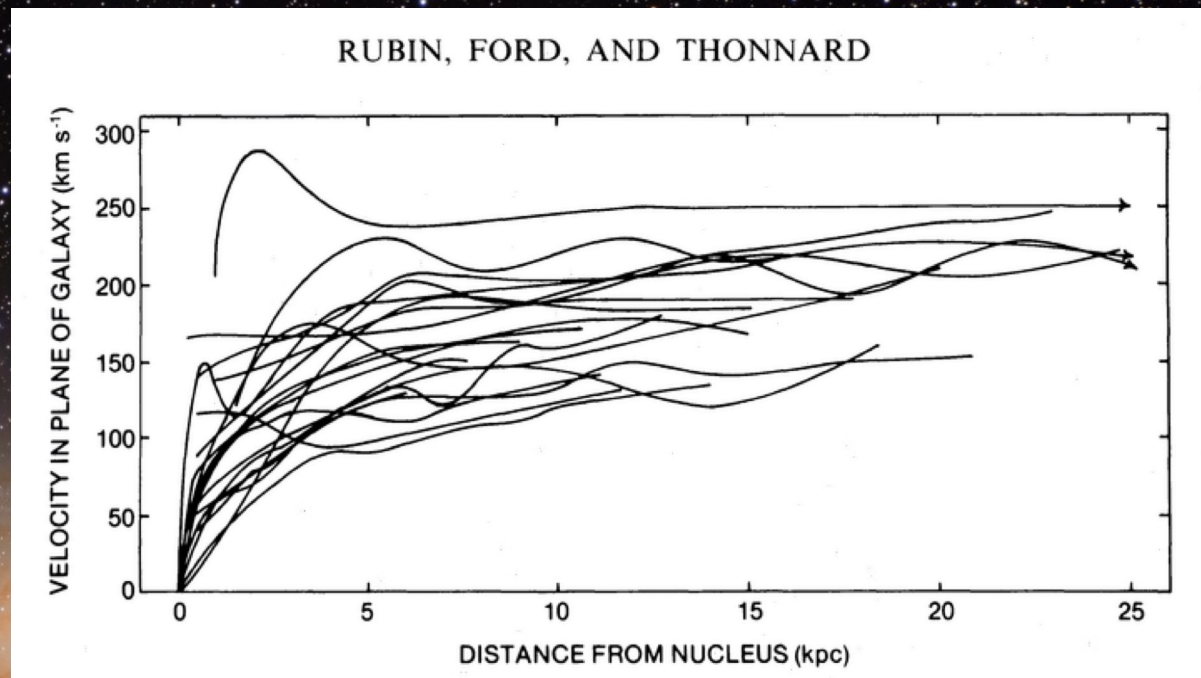
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DISTANCE

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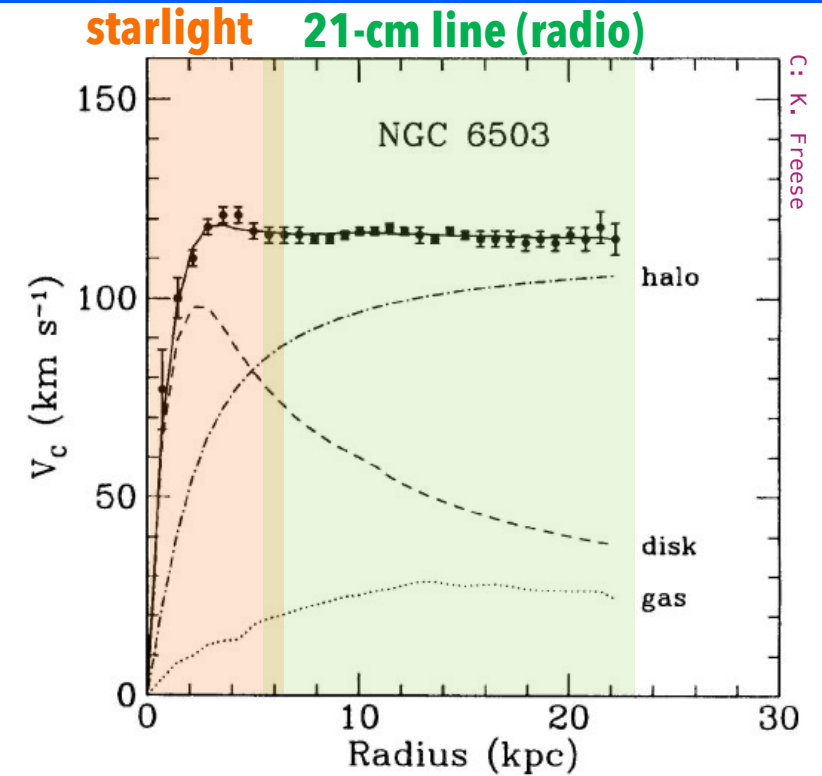
**Take-away messages from
galactic rotation curves**

What do rotation curves actually establish?



They measure the total gravitational field:

- detailed halo properties require a **multi-component fit** that comes with **its own set of assumptions** (e.g., stellar mass-to-light ratios, gas distribution, bulge and disk models, halo geometry and profiles, etc.)
- This is **crucial** in understanding the system, and also **very difficult** to do*



*more on this in **Lecture 2**.

Sofue & Rubin, ARA&A 39, 137 (2001), astro-ph/0010594; Salucci, A&A Rev. 27, 2 (2019), arXiv:1811.08843

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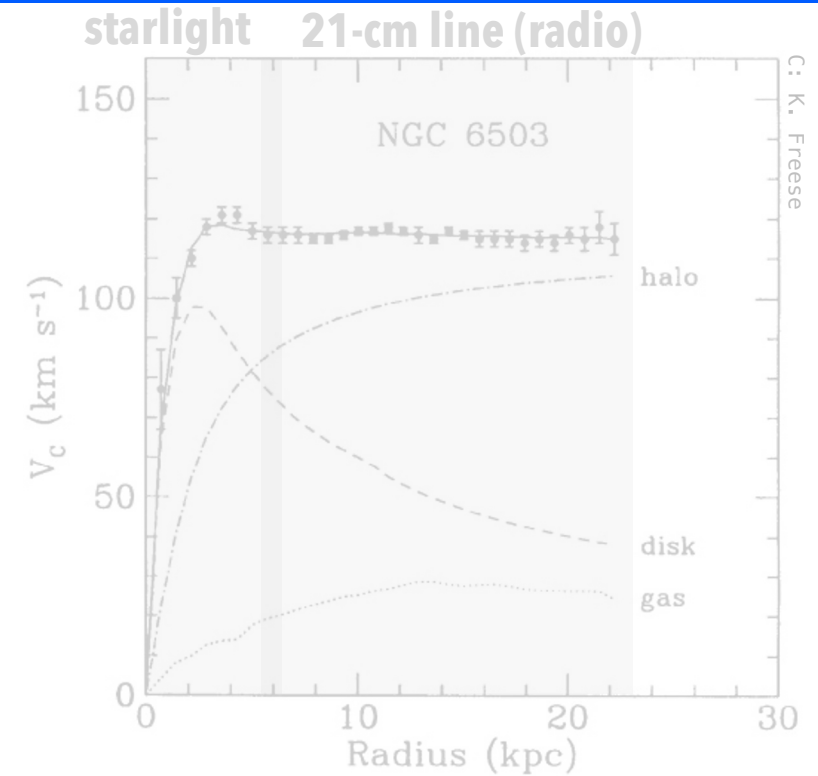
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Classical or new physics?

They alone do not determine what is the explanation (i.e., ordinary unseen matter; modified dynamics; a new particle, or something else).



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adjective: invisible, unseeable, etc.

dark matter



Ordinary, but non-luminous matter

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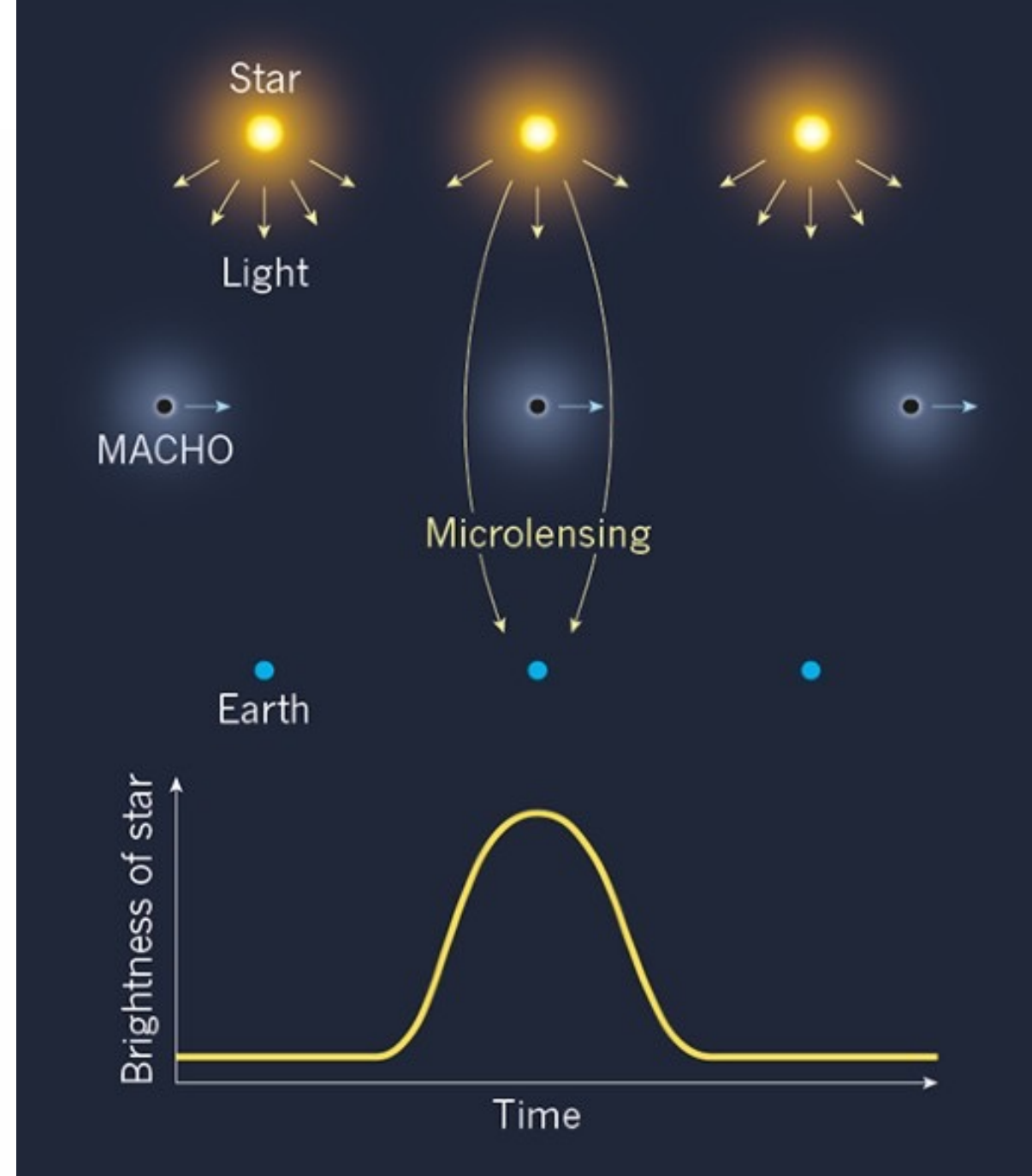
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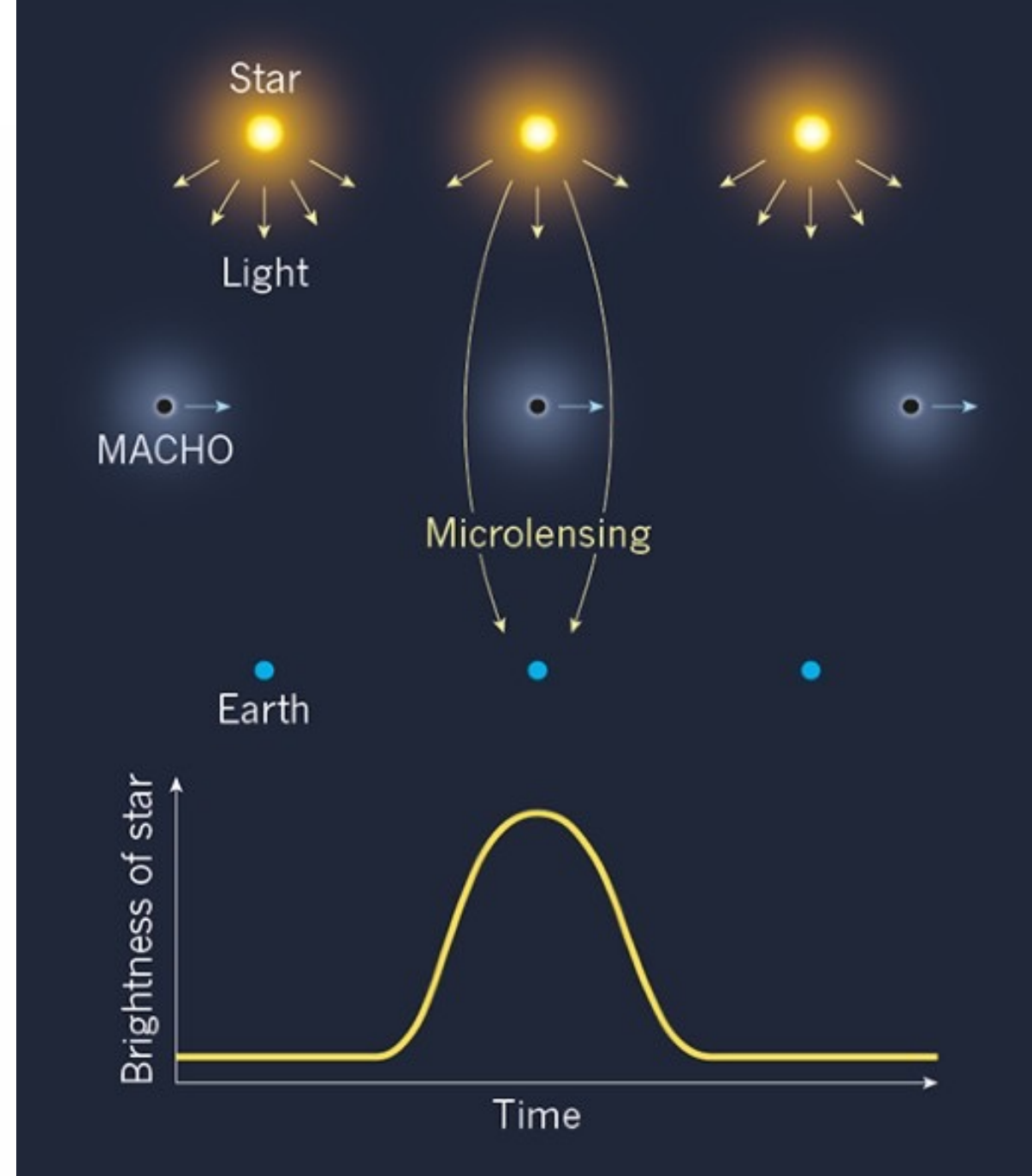
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- EROS-2 Survey (2007) reported an even smaller number of events (**one** candidate for **7 million bright stars** considered)

Microlensing rules out MACHOs as most of the Galactic dark matter over broad mass intervals.

Alcock et al. [MACHO Collab.], ApJ 542, 281 (2000), astro-ph/0001272;
Tisserand et al. [EROS-2], A&A 469, 387 (2007), astro-ph/0607207



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- Acceleration-dependent modifications to the Newton's 2nd law:

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- This is entirely empirical: it is considered an "emergent" theory, not fundamental.
- Fails if applied to other systems, and even more detailed data measurements with *Gaia*. [arXiv:2101.08374](#); [1812.08169](#)

Exercise 1.1

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MACHOs: Massive Compact Halo Objects

- faint low-mass stars, brown dwarfs, stellar remnants, planets and compact objects, cold or diffuse gas and dust
- The MACHO Collaboration analyzed **11.9 million stars**, with only **13-17 possible lensing events** detected
- EROS-2 Survey (2007) reported an even smaller number of events (**one** candidate for **7 million bright stars** considered)

Microlensing rules out MACHOs as most of the Galactic dark matter over broad mass intervals.

Alcock et al. [MACHO Collab.], ApJ 542, 281 (2000), astro-ph/0001272;
Tisserand et al. [EROS-2], A&A 469, 387 (2007), astro-ph/0607207

Modified dynamics

MOND: Modified Newtonian Dynamics

- Acceleration-dependent modifications to the Newton's 2nd law:

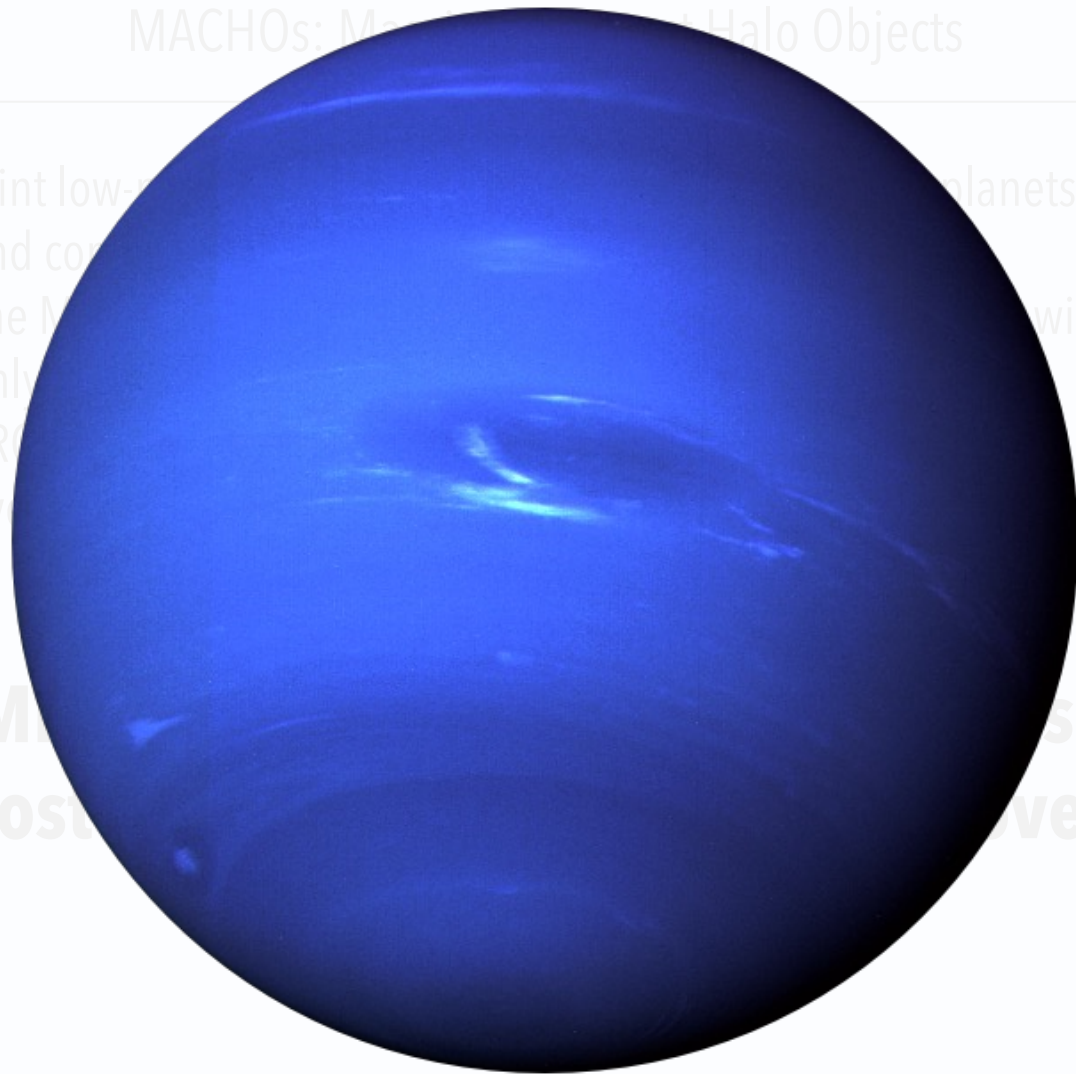
$$\mathbf{F} = m\mu\left(\frac{a}{a_0}\right)\mathbf{a} \quad \begin{array}{ll} \mu(x) \rightarrow 1; & x \gg 1 \quad \text{Newtonian limit} \\ \mu(x) \sim x; & x \ll 1 \quad \text{MOND regime} \end{array}$$

- This is entirely empirical: it is considered an "emergent" theory, not fundamental.
- Fails if applied to other systems, and even more detailed data measurements with *Gaia*. [arXiv:2101.08374](#); [1812.08169](#)

Exercise 1.1

Milgrom, ApJ 270, 365 (1983); for a review,
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Ordinary, but non-luminous matter



The discovery of Neptune through the characterization of Uranus' orbit

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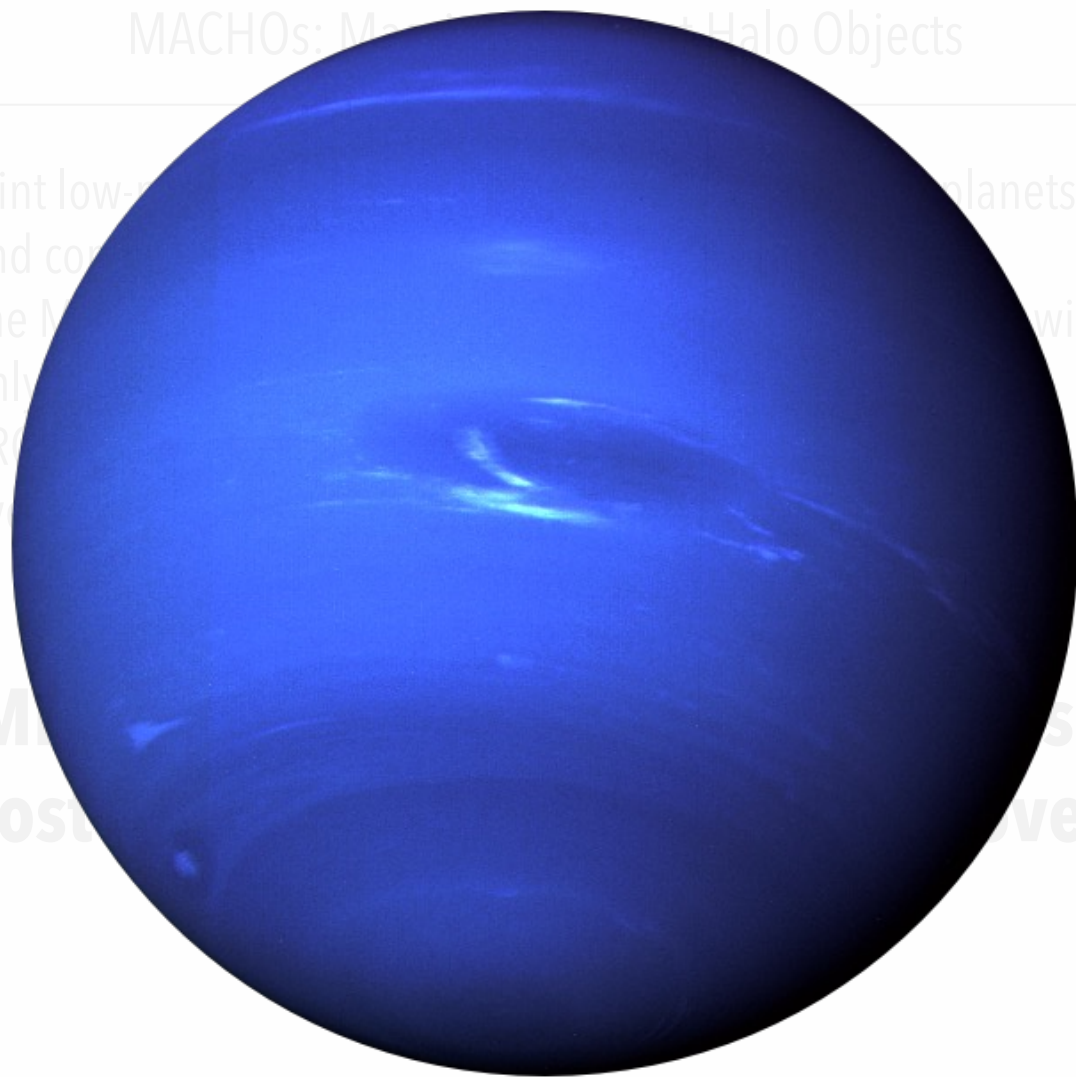
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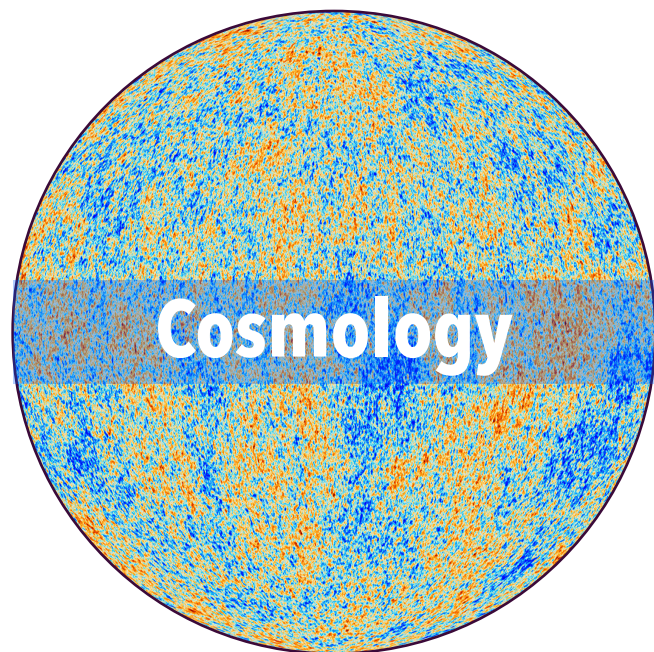


- Acceleration-dependent modifications to the Newton's 2nd law:
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Newtonian limit
MOND regime
 - This is entirely an "emergent" theory, not fundamental
 - Fails if applied to even more detailed data measurements with *Gaia*.
 - ...but remains a popular idea, albeit not amongst astrophysicists and particle physicists (see *APS' 2026 Big Mysteries Survey*, 2605.11058)
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- Describing Mercury's precession (missing 43"/century) needed GR

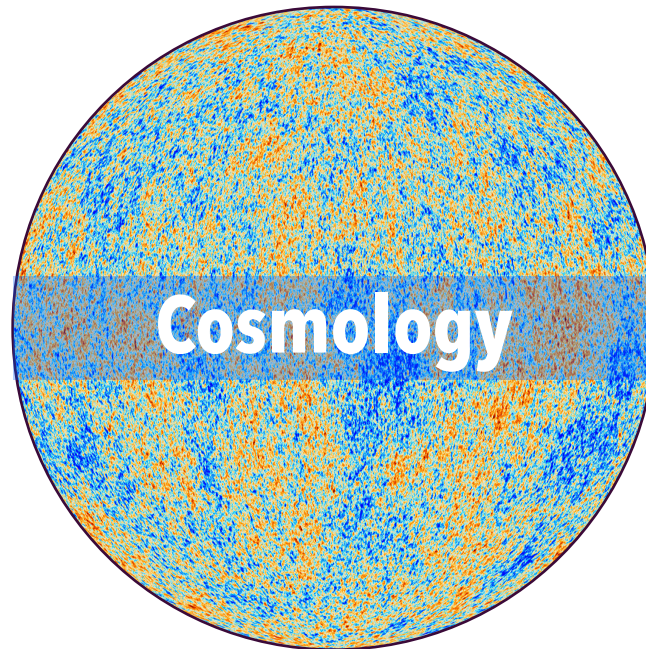
noun: component required to satisfy a specific set of observational constraints

dark matter



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Gravitational lensing: clusters revisited



Gravitational lensing: clusters revisited

Mass in the cluster *lenses* background galaxies.

Projected surface mass density Σ can be inferred from the deflection field $\hat{\alpha}$:

$$\hat{\alpha}(\xi) = \frac{4G}{c^2} \int \frac{(\xi - \xi')\Sigma(\xi')}{|\xi - \xi'|^2} d^2\xi'$$

Gravitational lensing

Gravitational lensing: clusters revisited

Assuming **hydrostatic equilibrium** of hot X-ray gas in the clusters allows us to trace out the mass distribution.

$$\frac{d\Phi}{dr} = \frac{GM_{\text{tot}}(< r)}{r^2} = -\frac{1}{\rho_{\text{gas}}} \frac{dP_{\text{gas}}}{dr}$$

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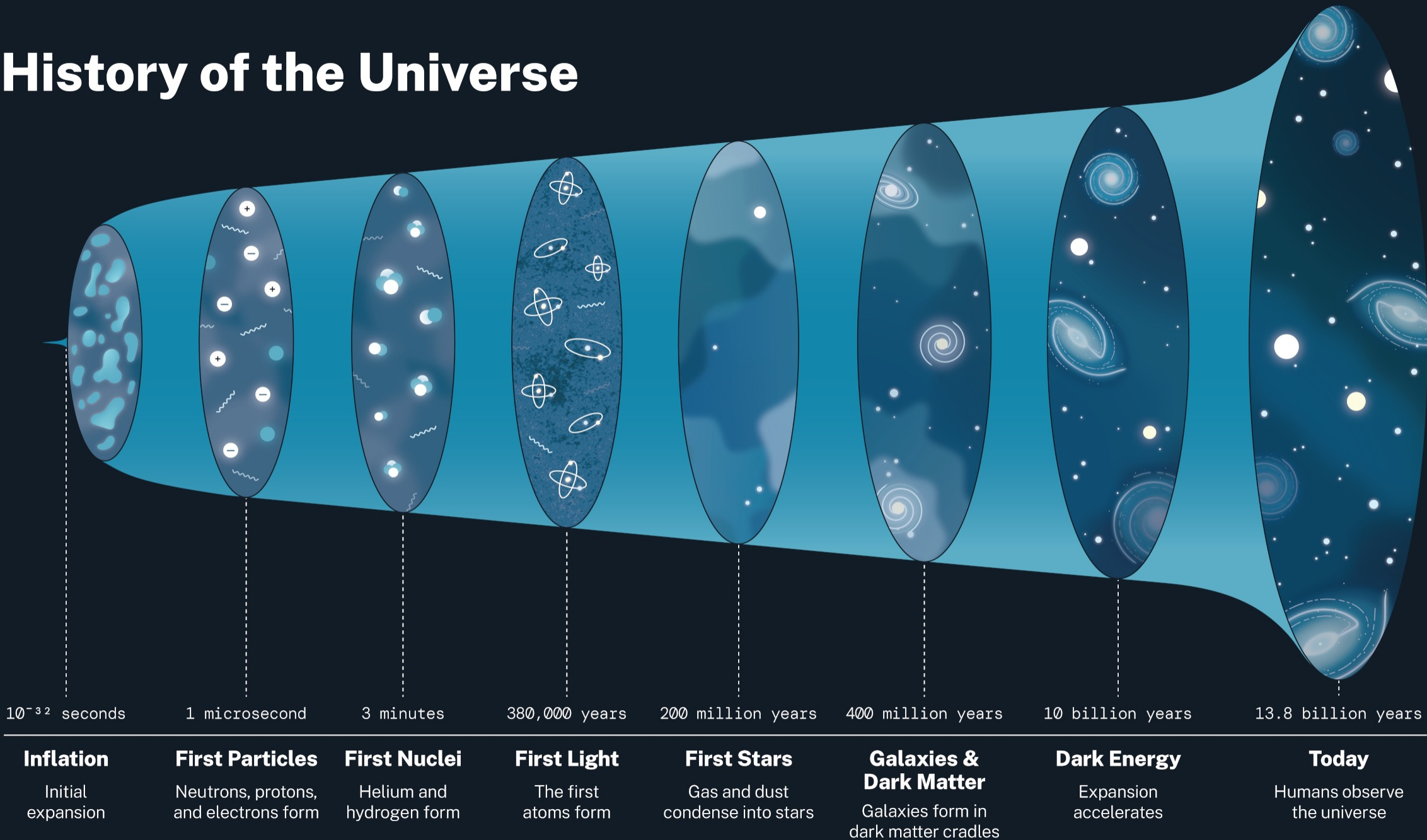
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Massey, Kitching & Richard, Rep. Prog. Phys. 73, 086901 (2010), arXiv:1001.1739 [lensing];
Lewis, Buote & Stocke, ApJ 586, 135 (2003) [X-ray mass]

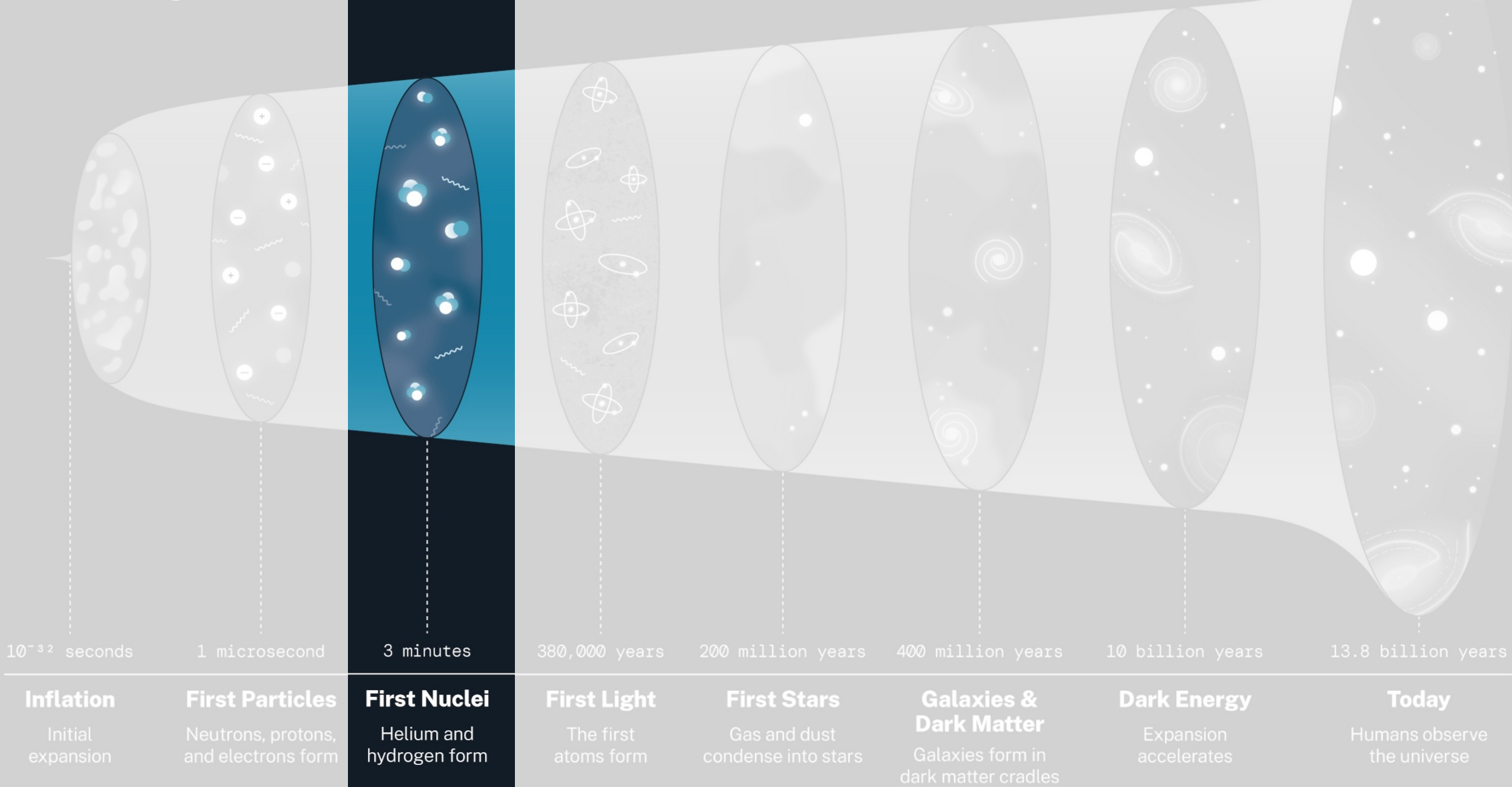
X-rays

Gravitational lensing

History of the Universe

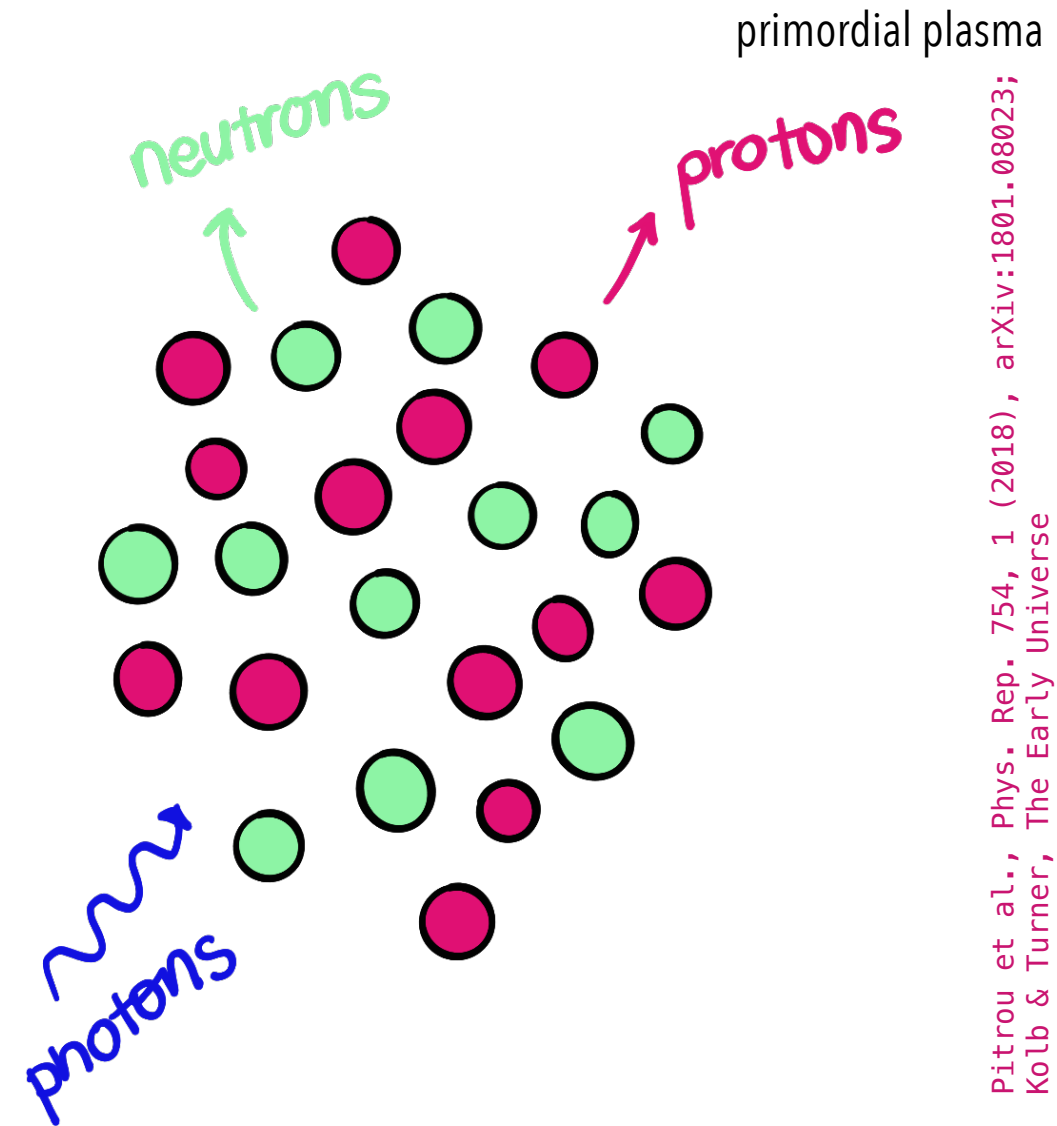


History of the Universe



Big Bang Nucleosynthesis (BBN)

- Early Universe: hot, dense plasma ($T \sim 1$ MeV)
- Free protons, neutrons, and huge bath of photons ($\sim 10^9$ photon per baryon)



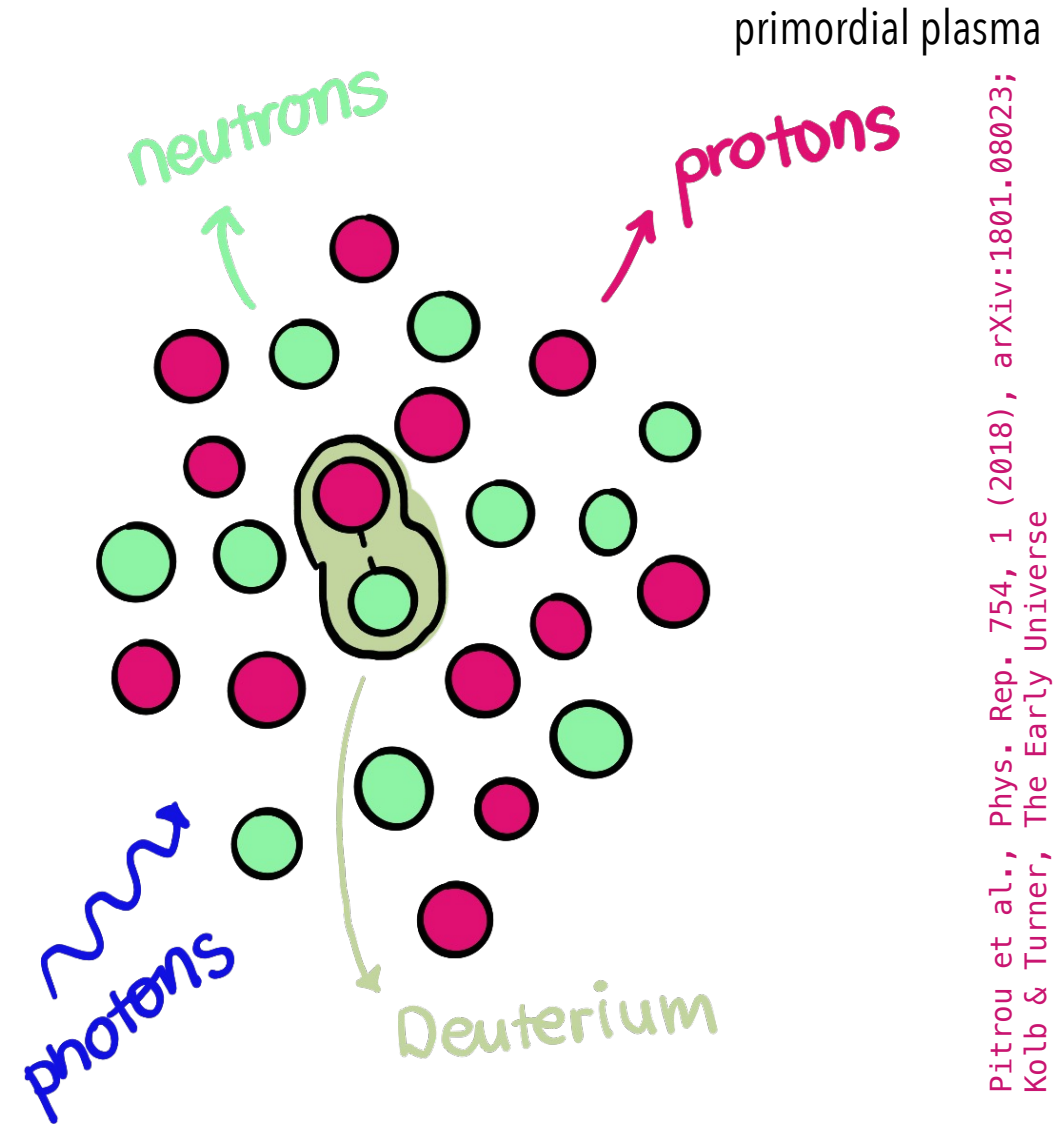
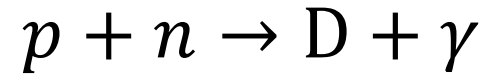
Pitrou et al., Phys. Rep. 754, 1 (2018), arXiv:1801.08023;
Kolb & Turner, The Early Universe

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...tells us how much baryonic (ordinary) matter is there in the Universe, regardless of whether it's luminous or not.

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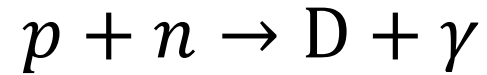
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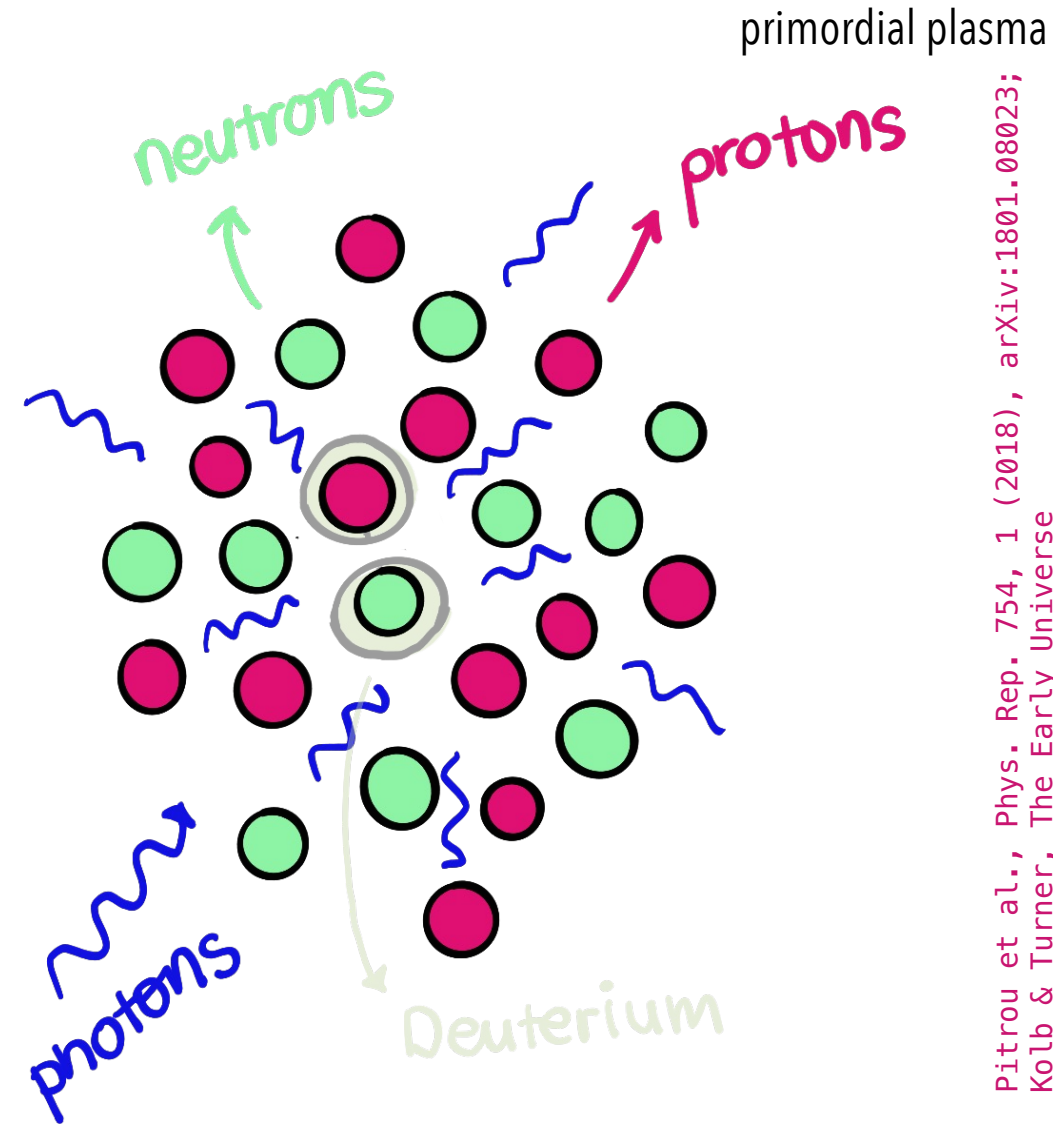
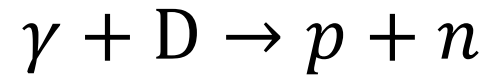
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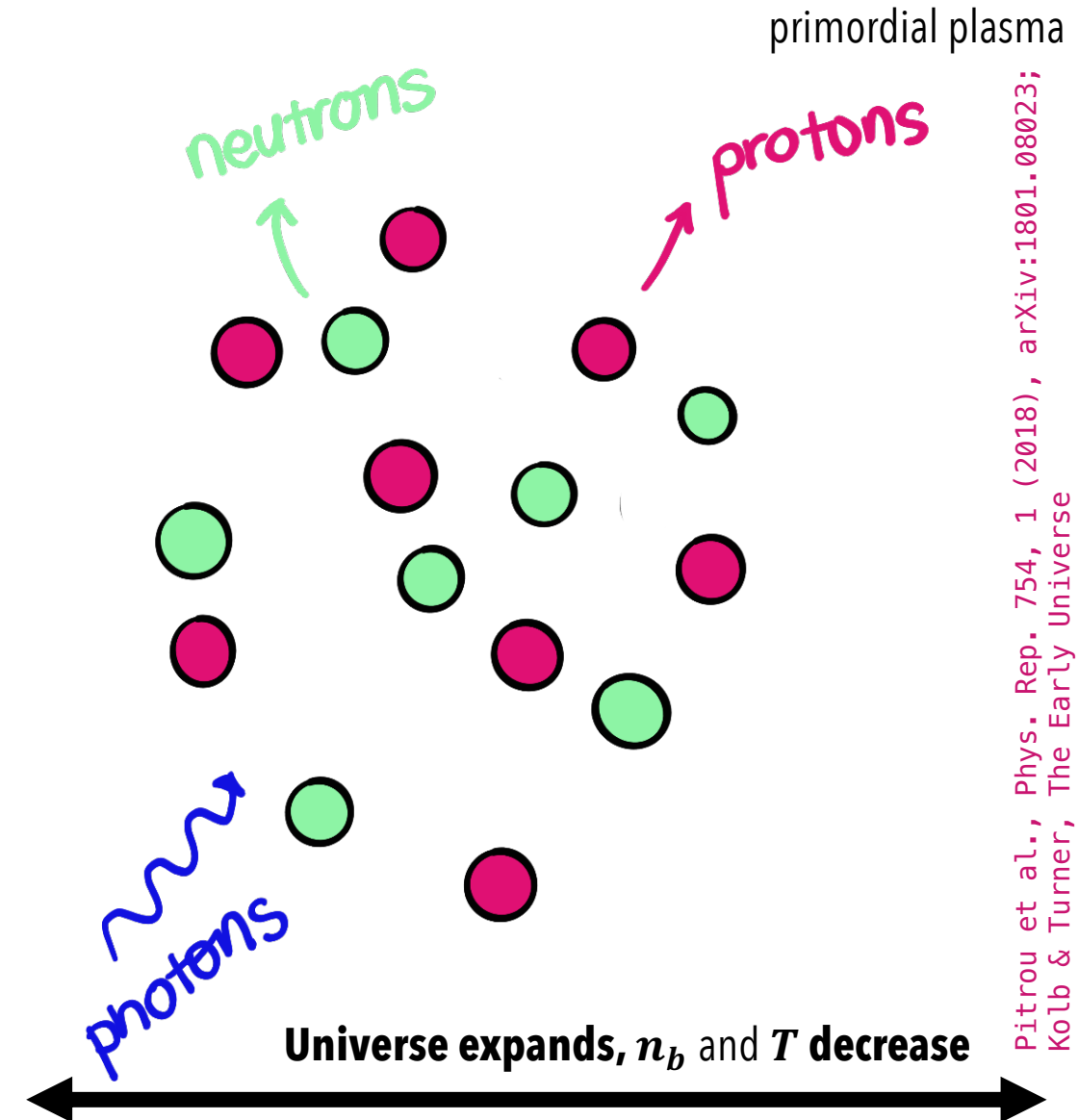


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Big Bang Nucleosynthesis (BBN)

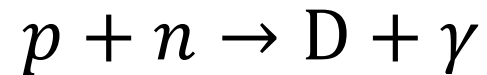
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$$p + n \rightarrow D + \gamma$$
- D cannot survive yet,
$$\gamma + D \rightarrow p + n$$
- Universe expands, temperature & density drop



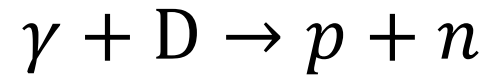
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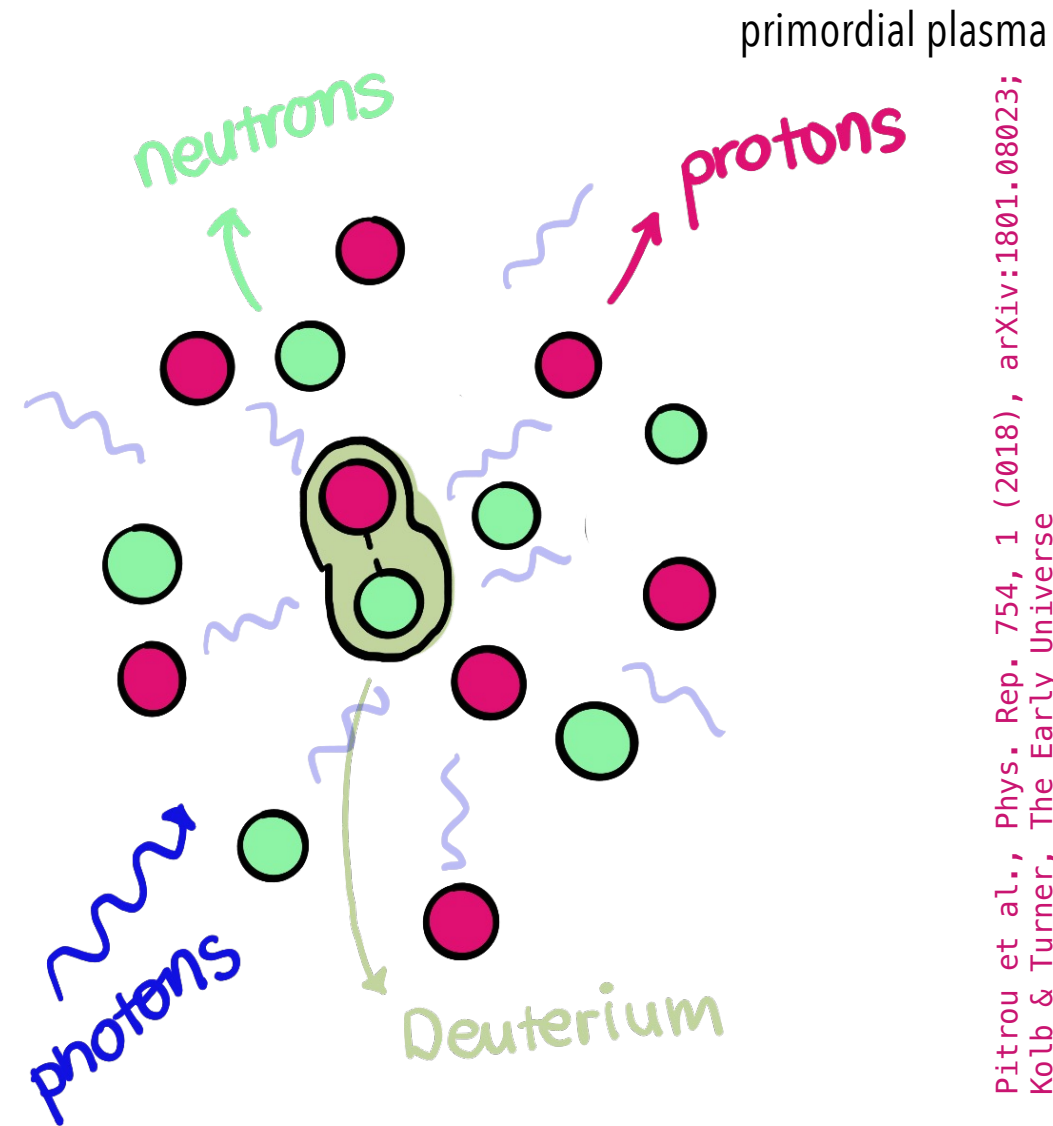


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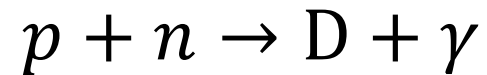
- **At $T \sim 0.1$ MeV, Deuterium survives!**



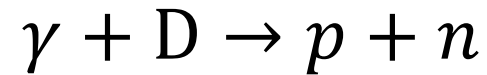
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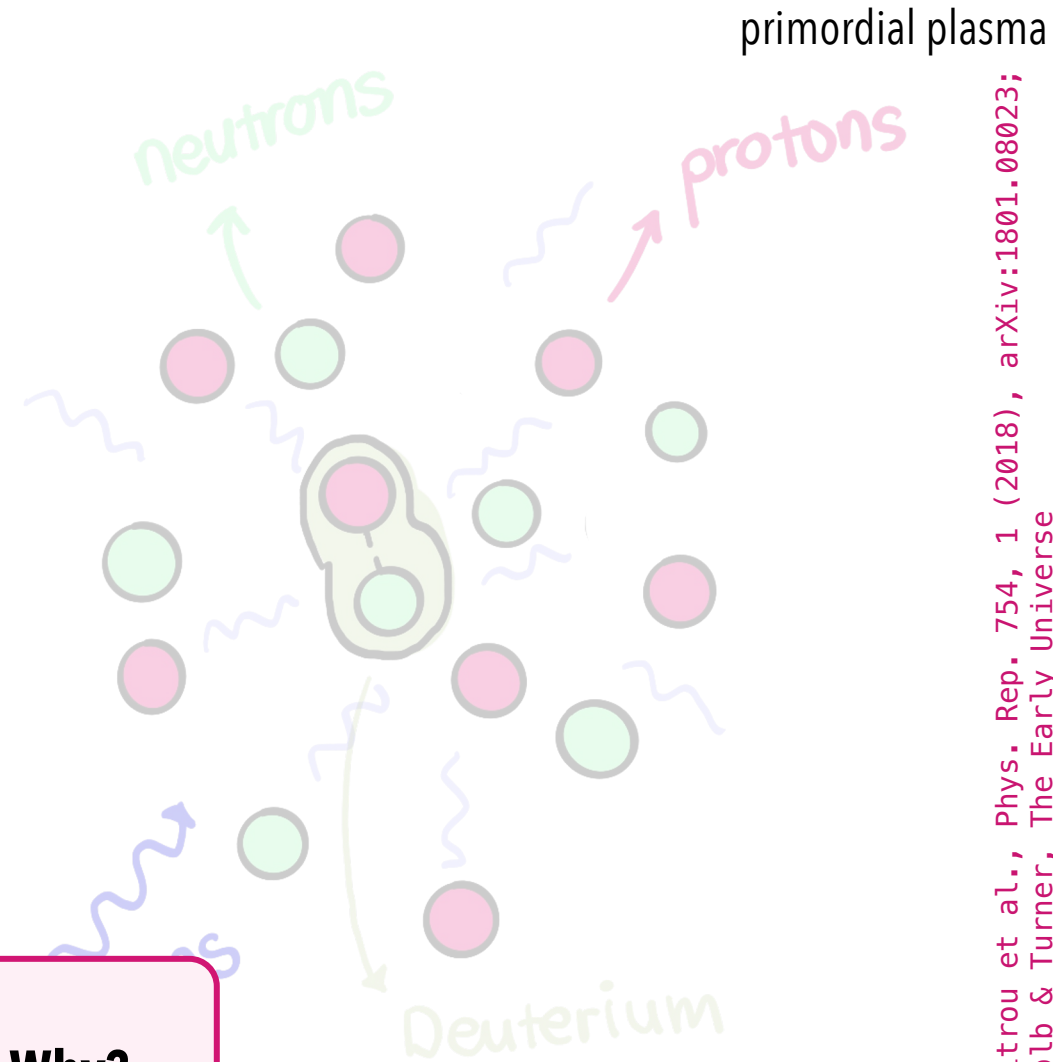


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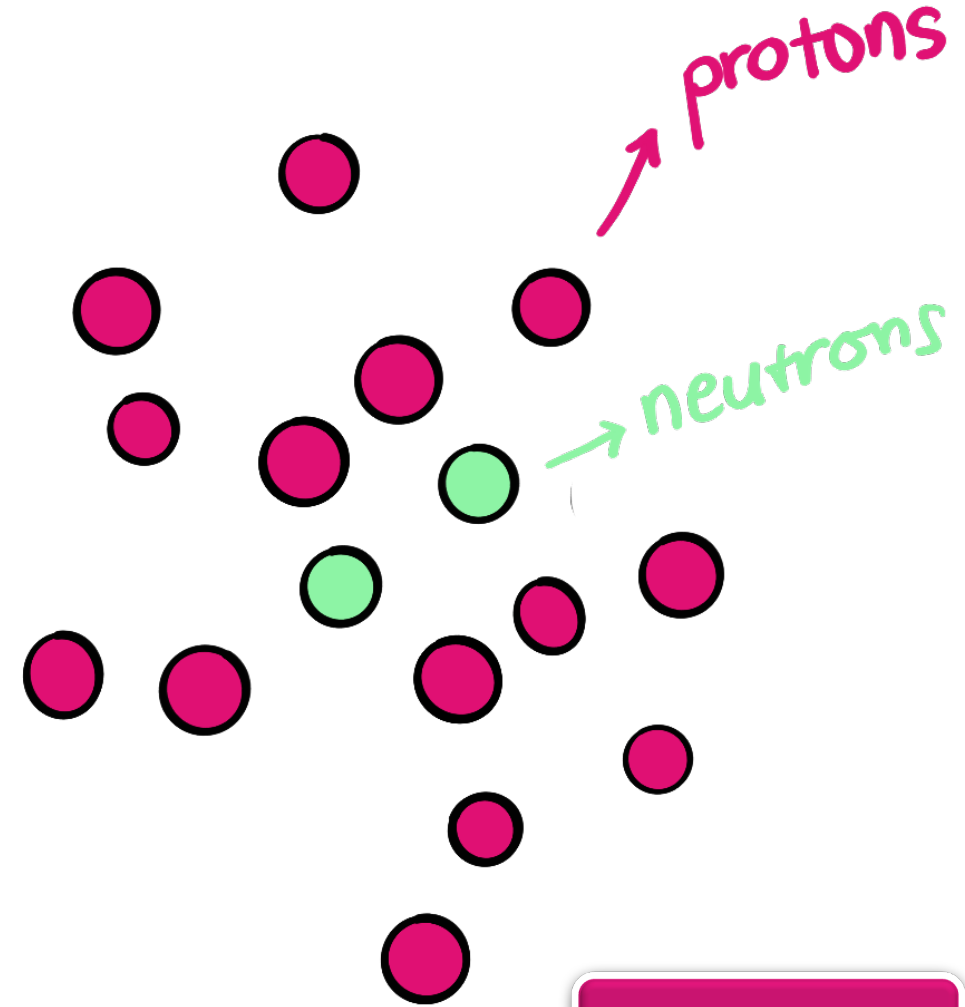
Q1. Deuterium has a binding energy of $B_D \sim 2.2 \text{ MeV}$.

Yet, we have to wait until $T \sim 0.1 \text{ MeV}$ so that it doesn't photodisassociate. **Why?**



Big Bang Nucleosynthesis (BBN)

- From there,
 $p, n \rightarrow \text{D}, {}^3\text{He}, {}^4\text{He}, {}^7\text{Li}$
- By the time the nucleosynthesis gets going, roughly
 $n/p = 1/7^*$

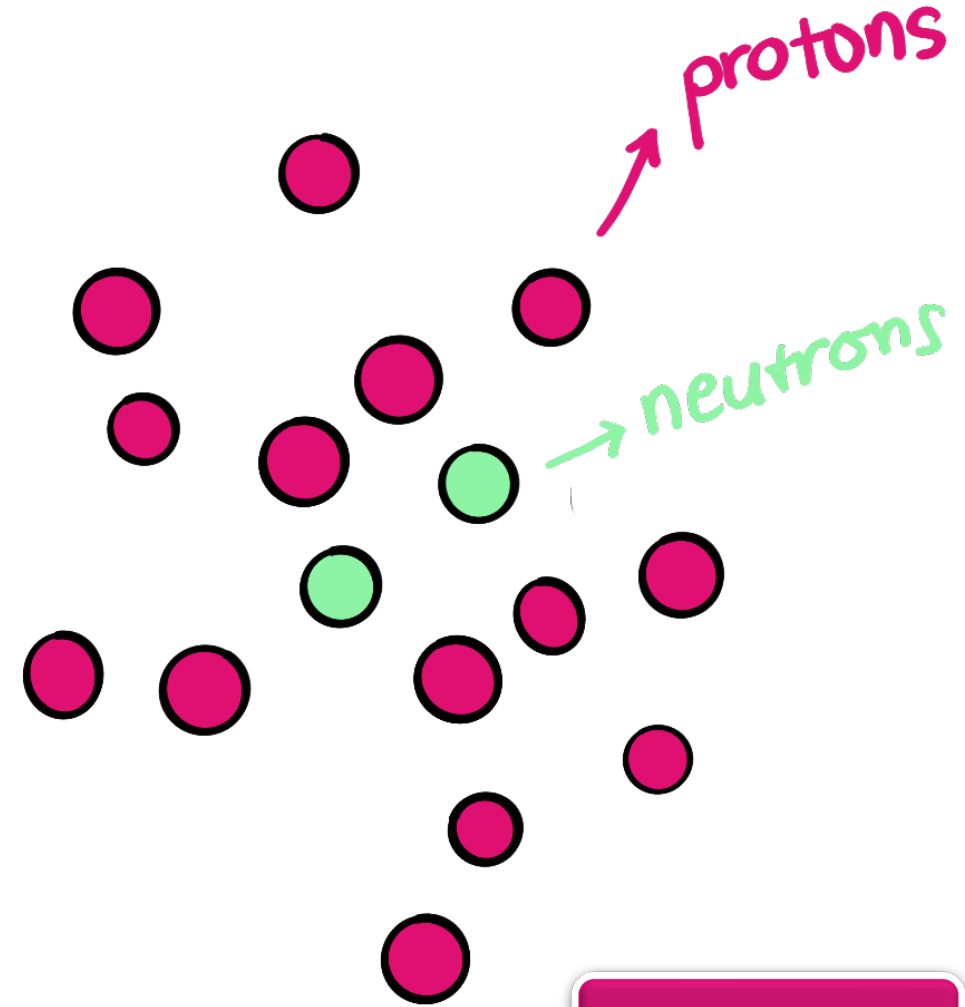


***Exercise 1.2**

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Most neutrons end up in ${}^4\text{He} \rightarrow$
He abundance is controlled by **how many neutrons were available** when the nucleosynthesis began



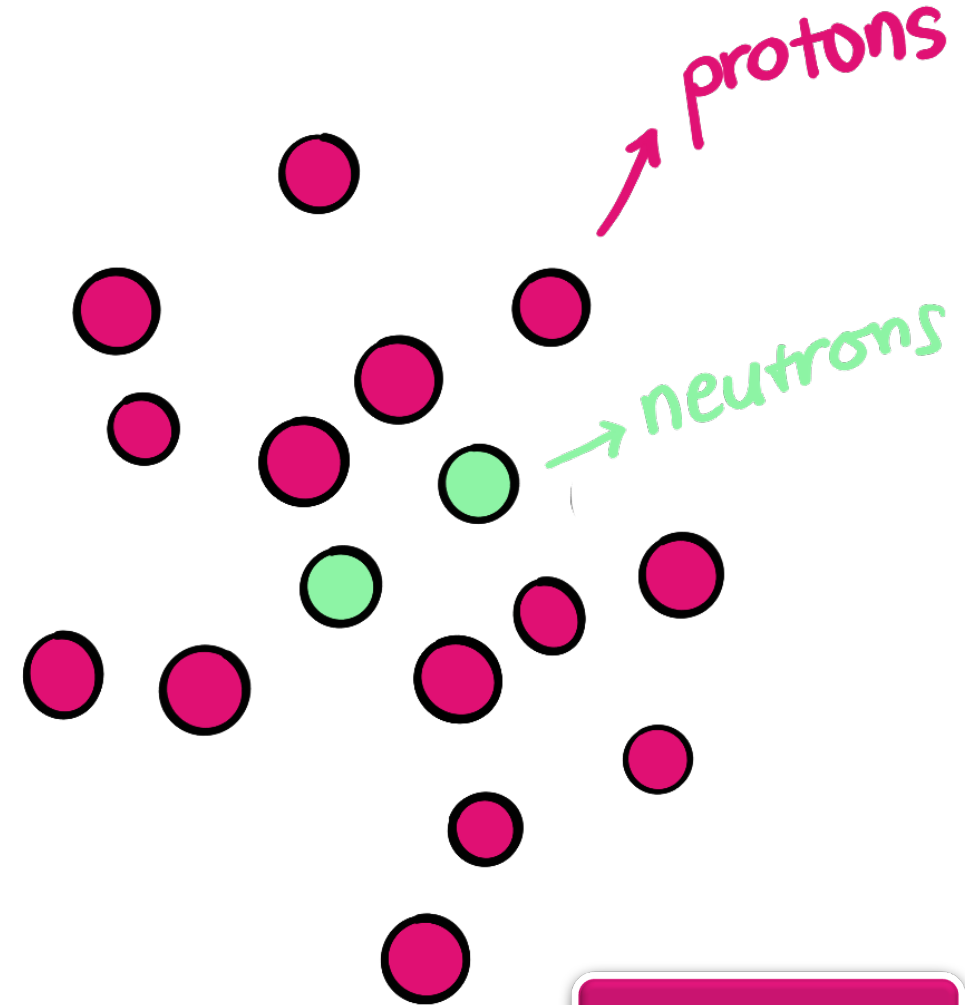
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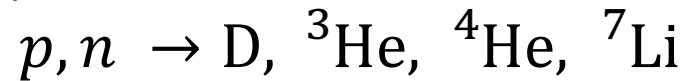
Leftover D abundance: controlled by how **efficient the nuclear processes** were, $\Gamma_{\text{nuc}} = n_b \langle \sigma v \rangle$



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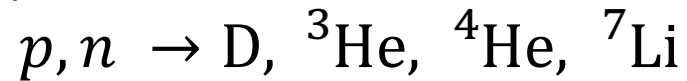
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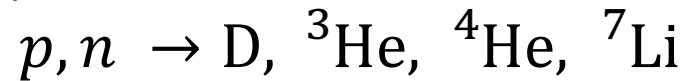
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$n_b \uparrow \rightarrow$ more nuclear reactions $\rightarrow$ more D processed into ${}^4\text{He} \rightarrow D/H \downarrow$

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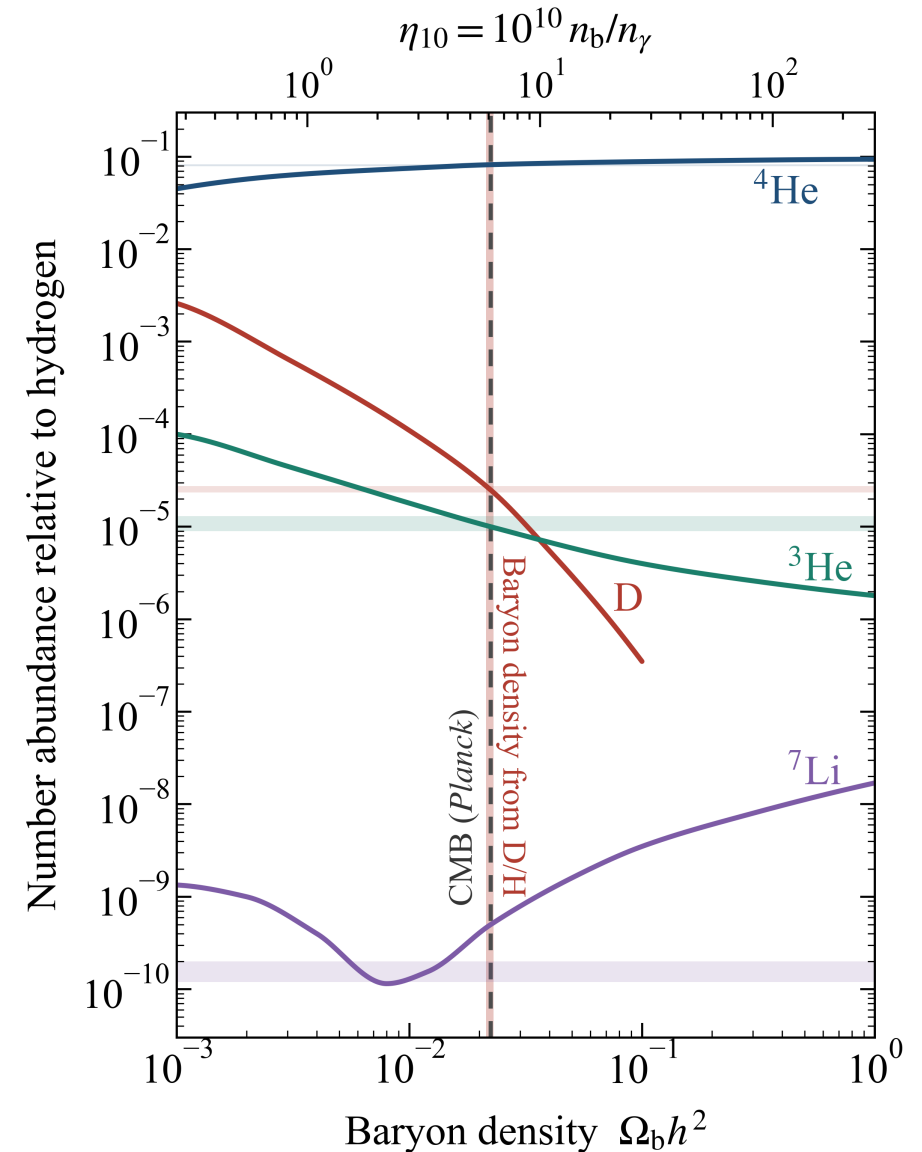
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$$\omega_b \equiv \Omega_b h^2 \sim 0.022.$$

Or, about 5% of the Universe's energy budget.

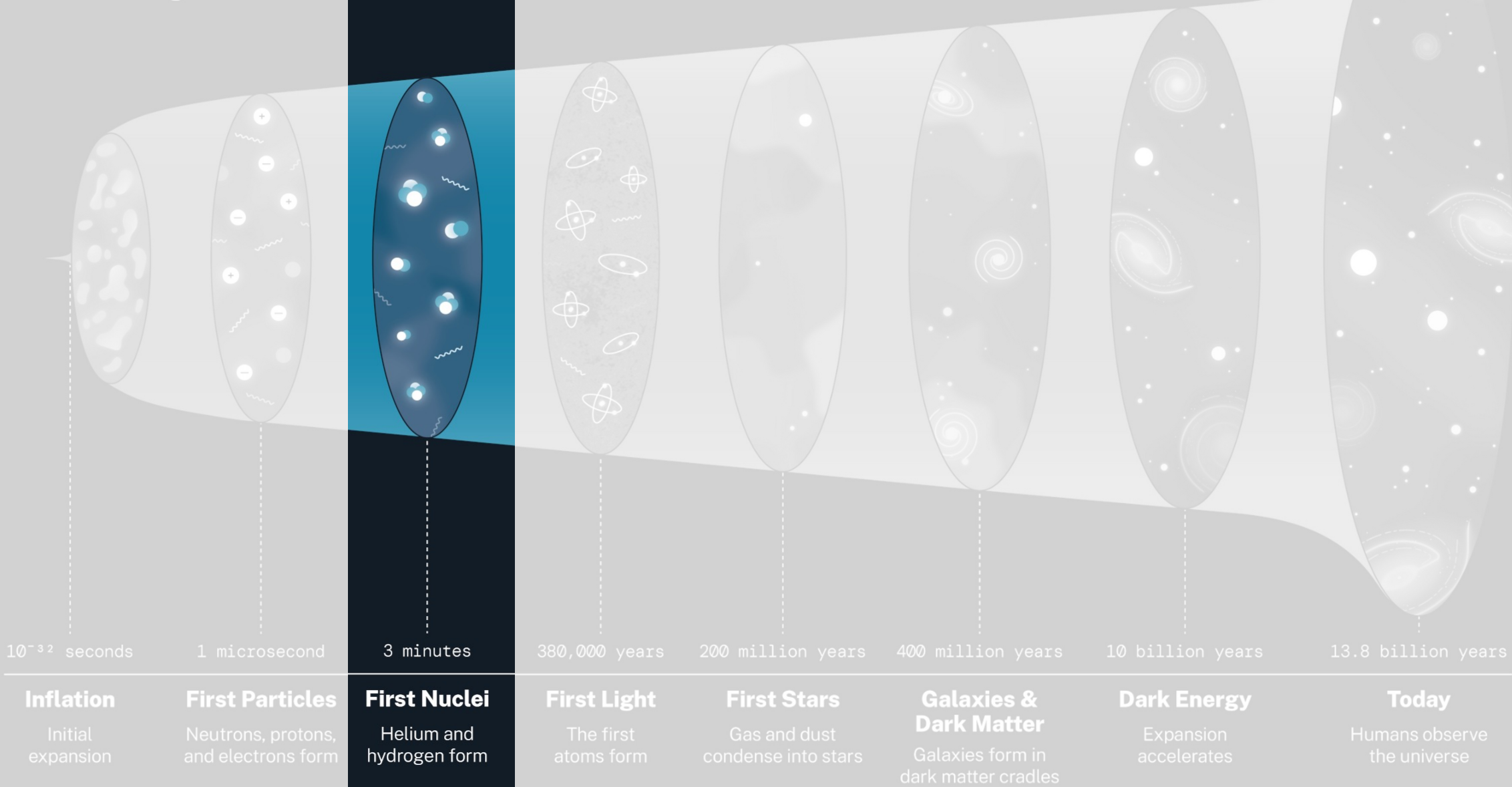


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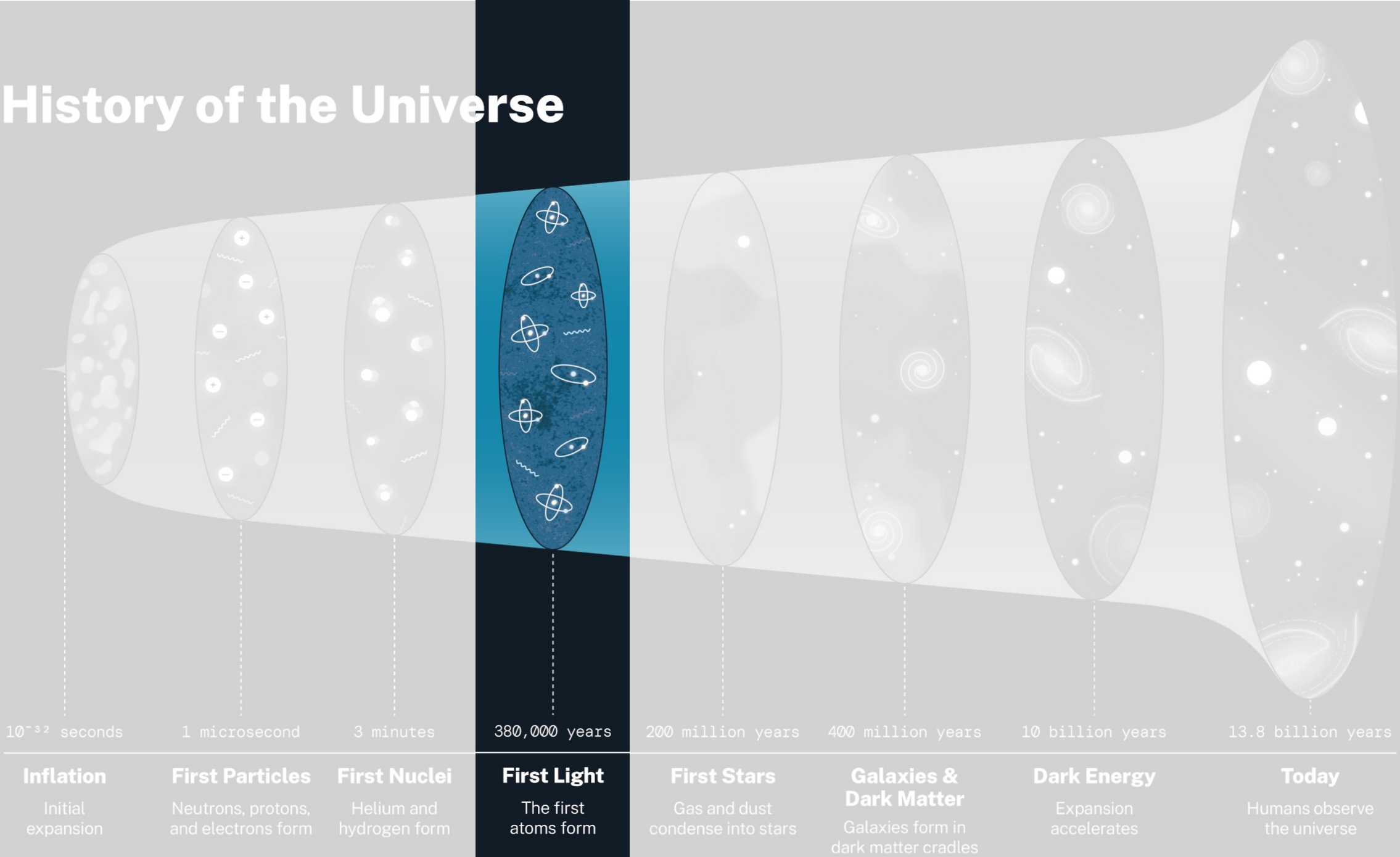
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History of the Universe

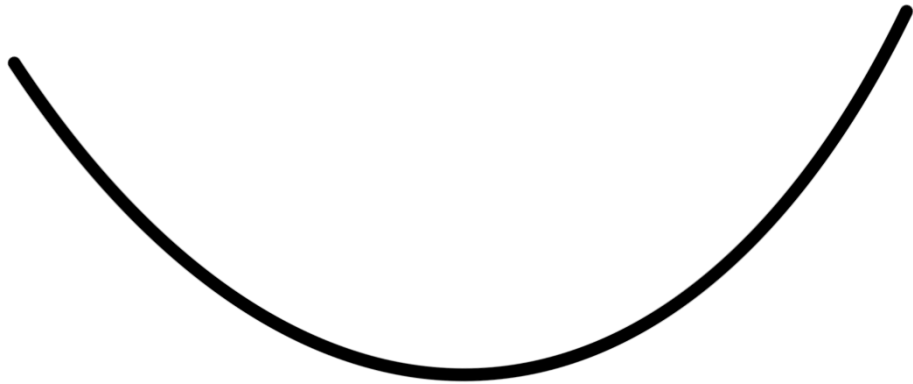


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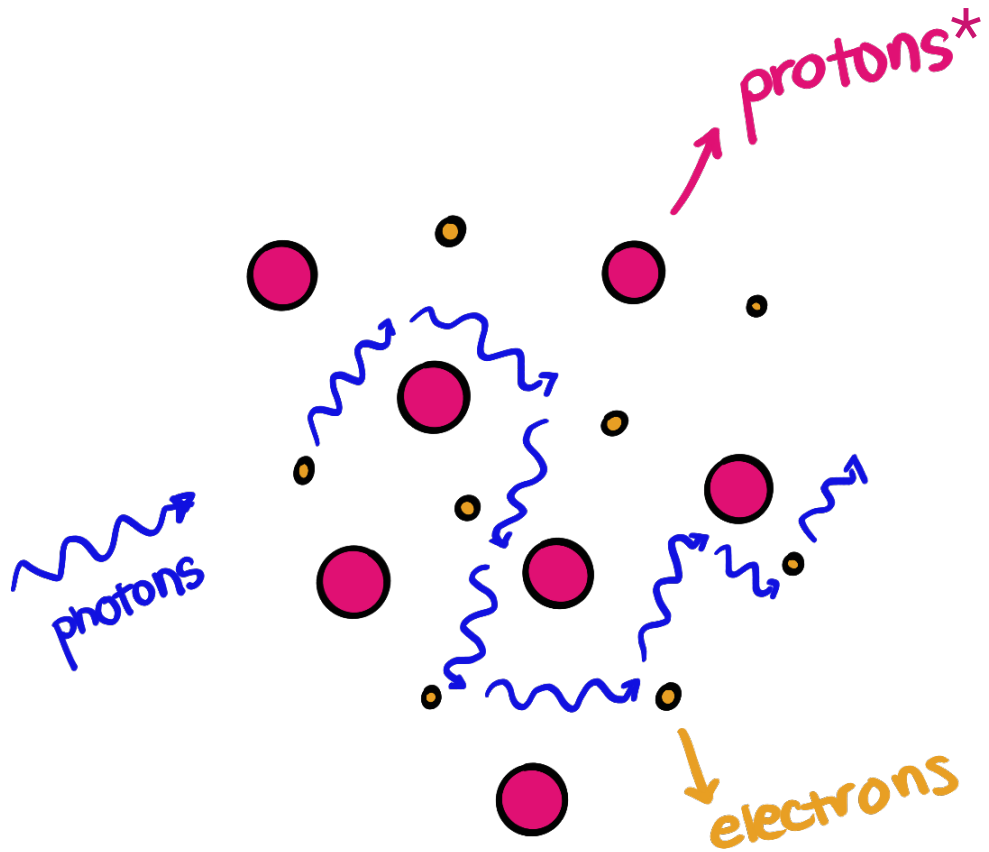


Cosmic Microwave Background (CMB)

- Primordial curvature fluctuations $\sim 10^{-5}$
seed gravitational wells of different sizes



Cosmic Microwave Background (CMB)

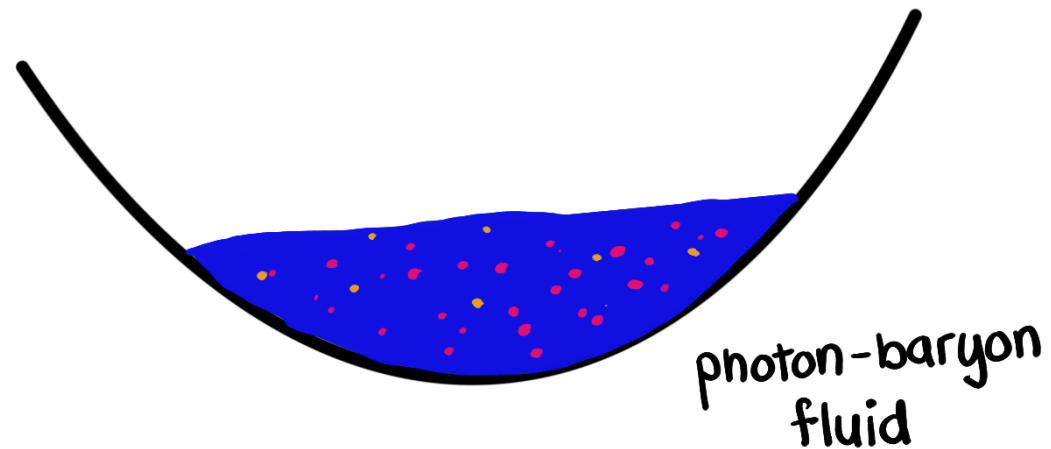


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- Photons Thomson-scatter from free electrons

$$\gamma + e^{-} \leftrightarrow \gamma + e^{-}$$

→ photon+baryons act as tightly coupled fluid

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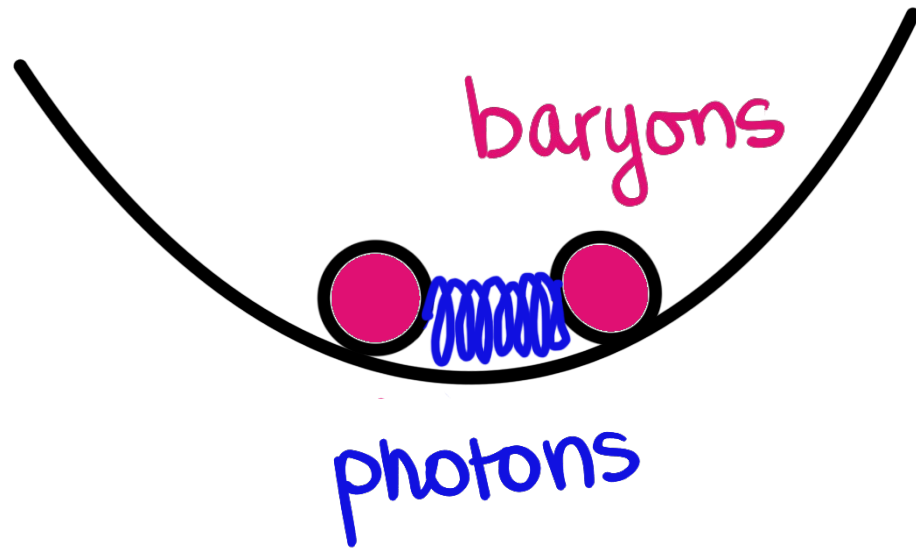
→ photon+baryons act as tightly coupled fluid

- The photon-baryon fluid fills the potential wells

Cosmic Microwave Background (CMB)

Analogy:

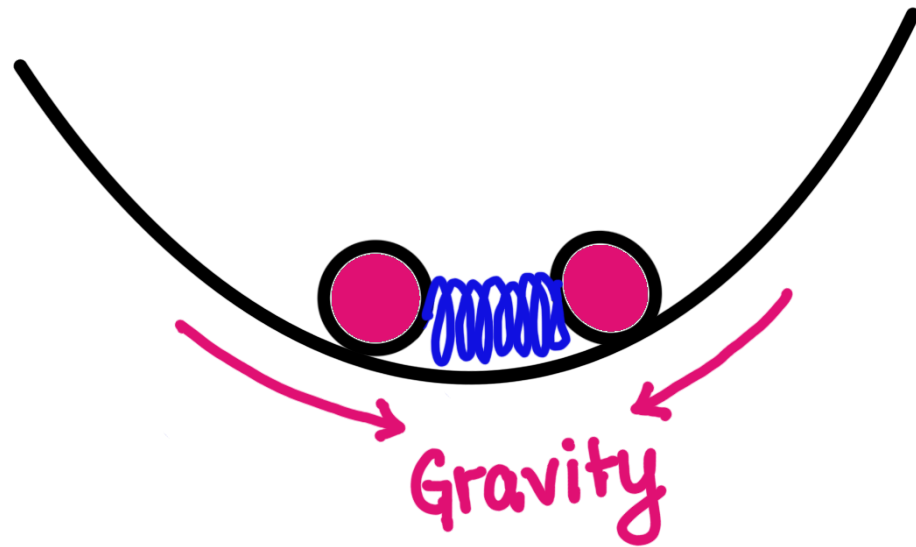
- **Pink balls:** baryonic inertia
- **Blue spring:** photon pressure
- **Well:** gravitational potential



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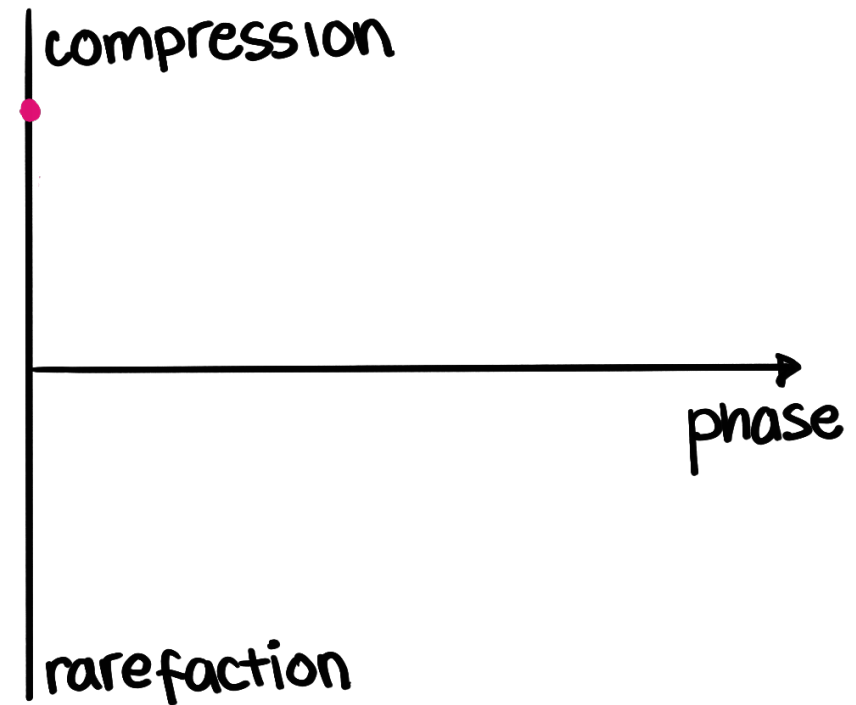
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Initial overdensity.

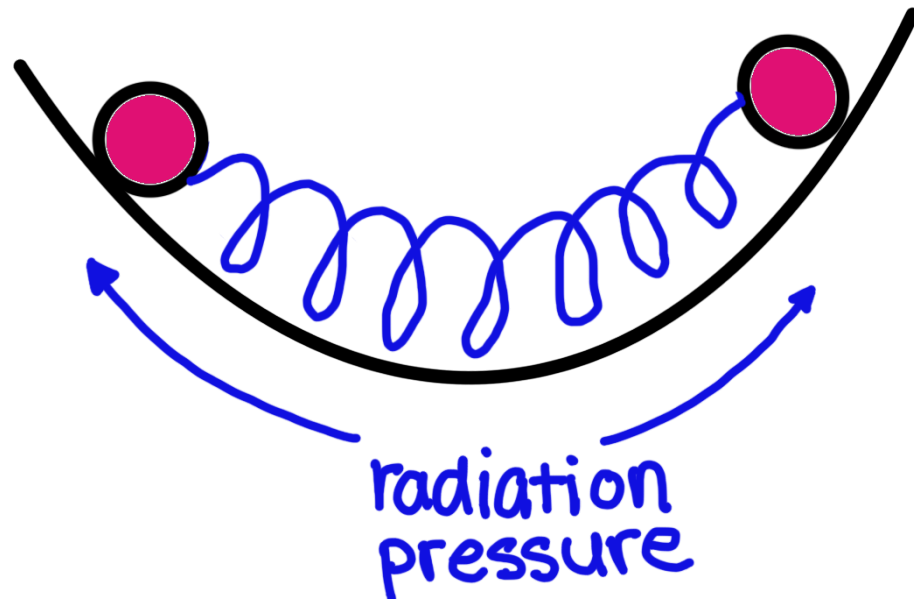
Fluid density $\uparrow$
Radiation pressure $\uparrow$



Cosmic Microwave Background (CMB)

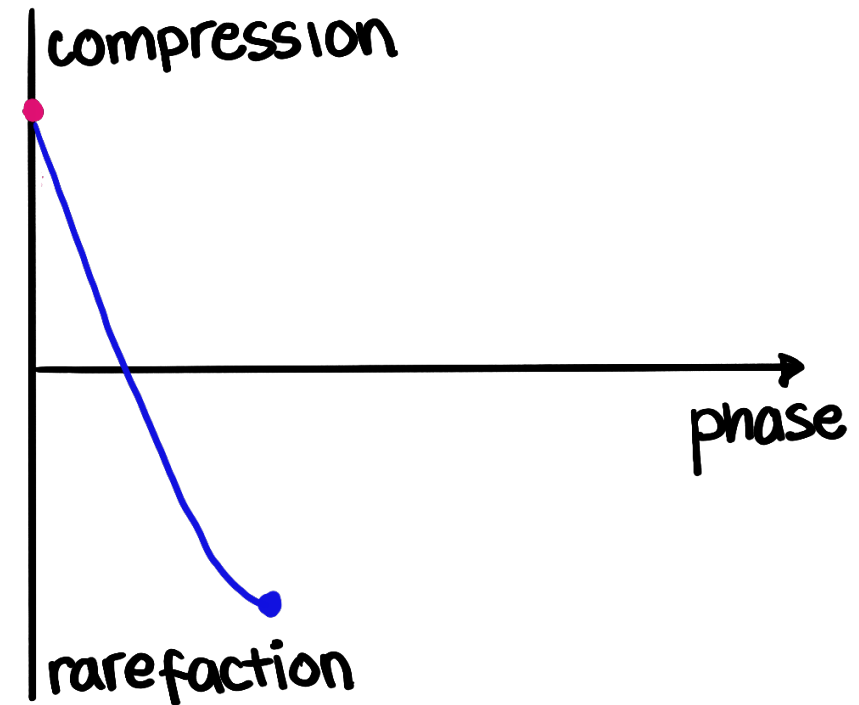
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Radiation pressure pushes the fluid outward & overshoots equilibrium driving rarefaction.

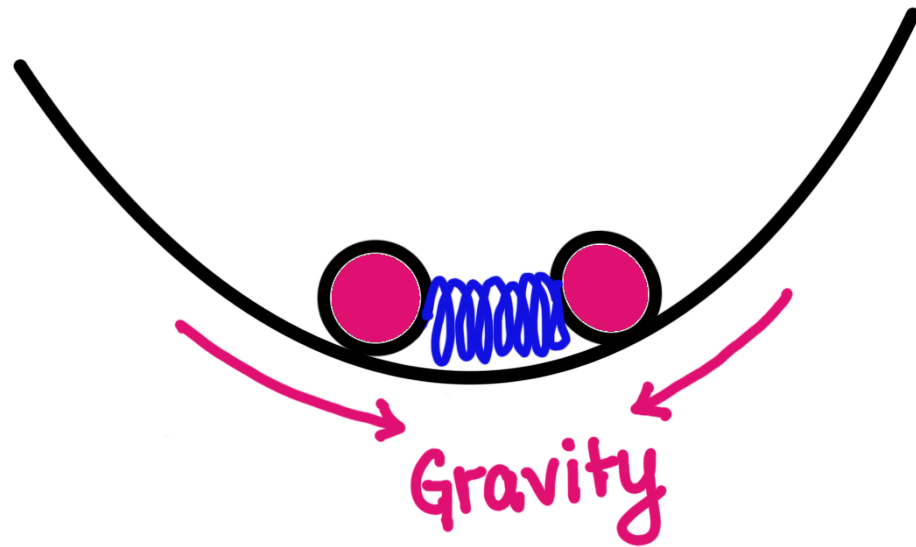
Gravity pulls it back.



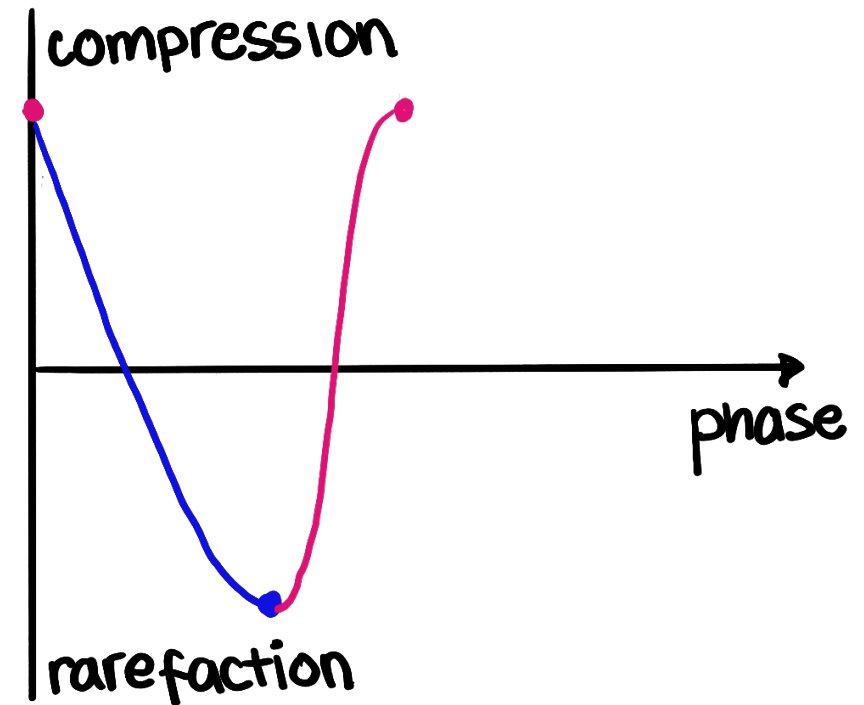
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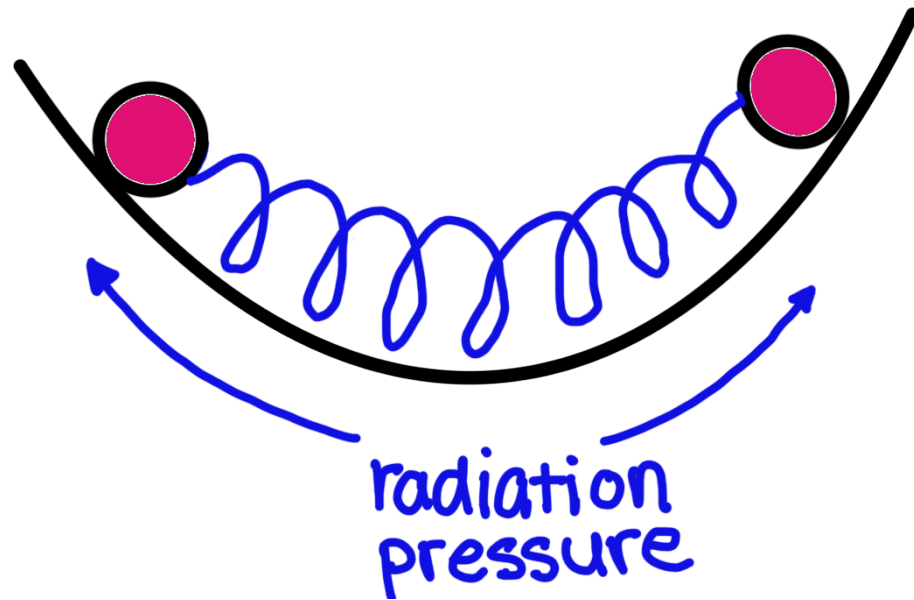
One full acoustic oscillation.



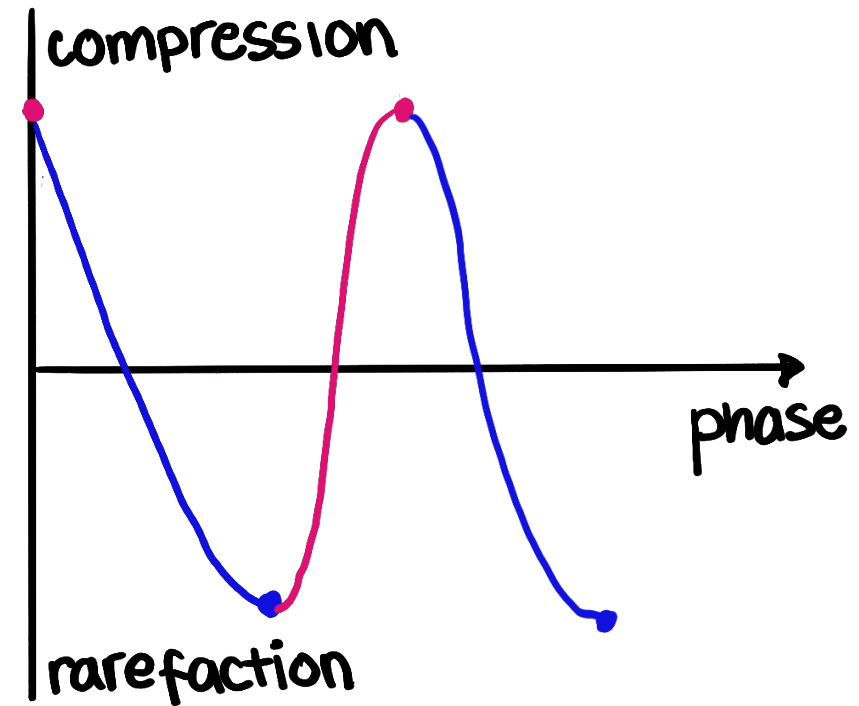
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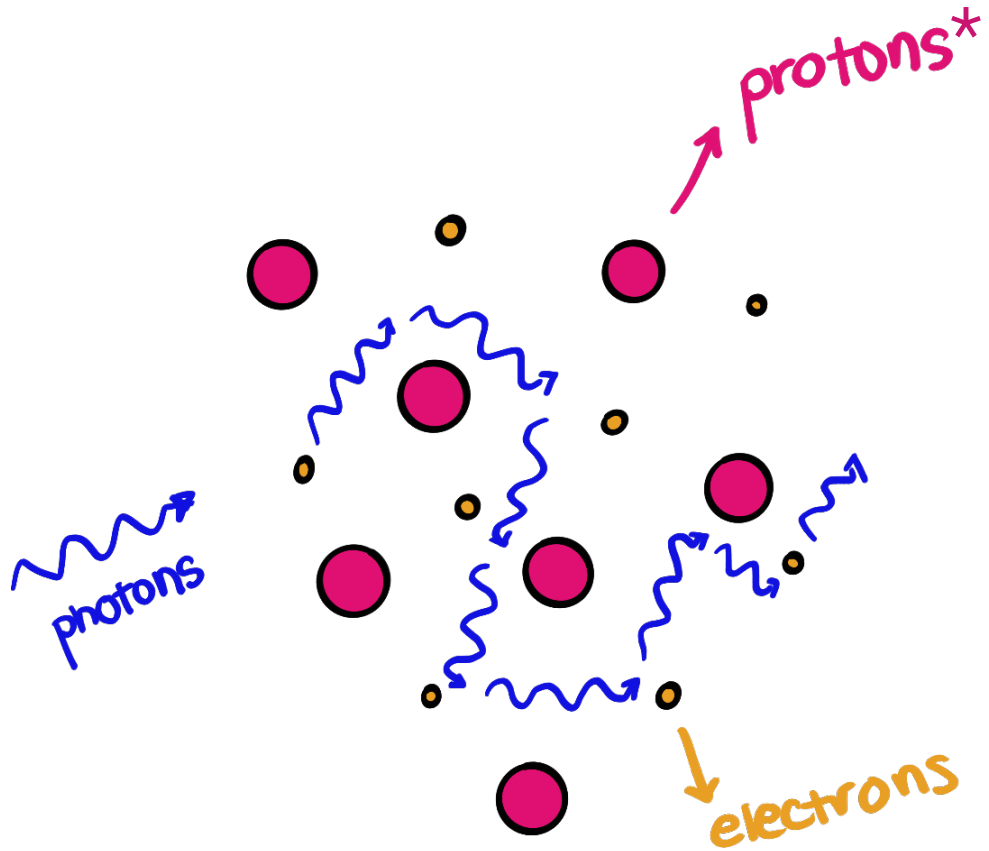
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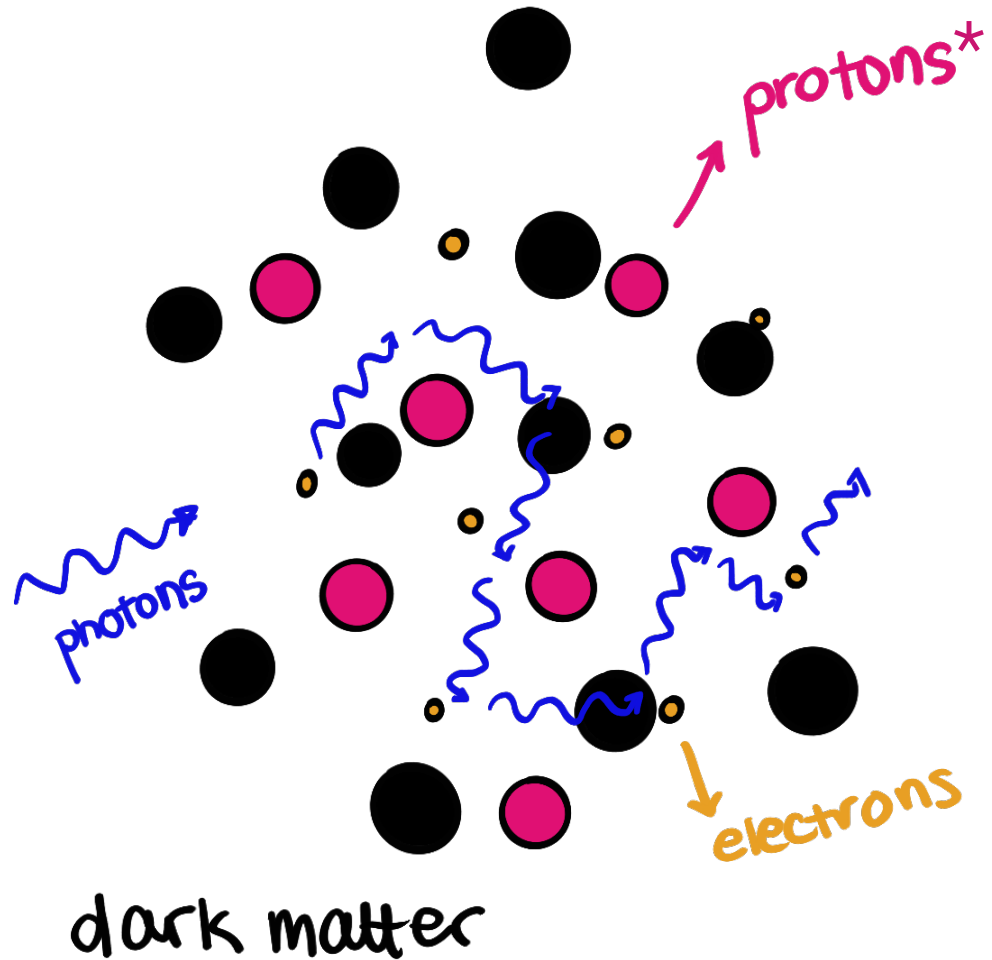
Continues...



Cosmic Microwave Background (CMB)



Cosmic Microwave Background (CMB)



Introduce **dark matter**, which:

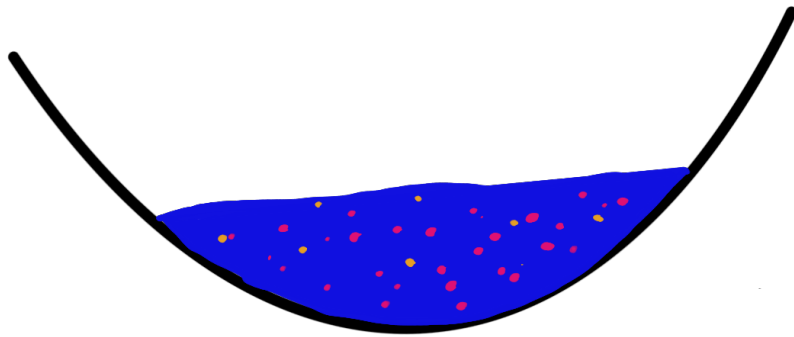
- is **not** part of the baryon-photon fluid
 - does **not** feel radiation pressure
- does not participate in the acoustic oscillations

Cosmic Microwave Background (CMB)

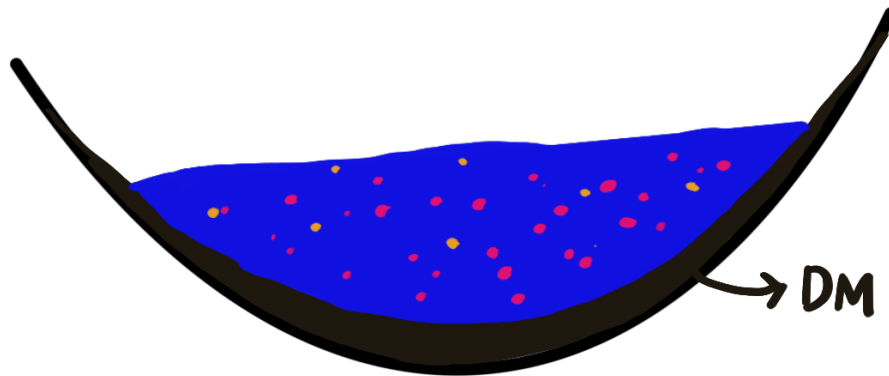
CDM?

Time t_1 (horizon crossing)

No



Yes



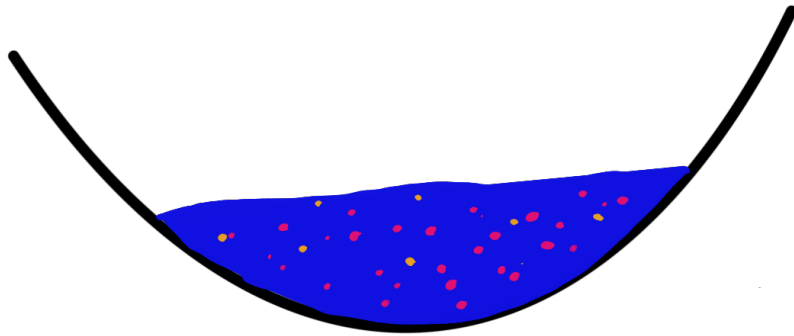
Cosmic Microwave Background (CMB)

CDM?

Time t_1 (horizon crossing)

Time $t_2 > t_{\text{eq}}$ (after matter-radiation equality)

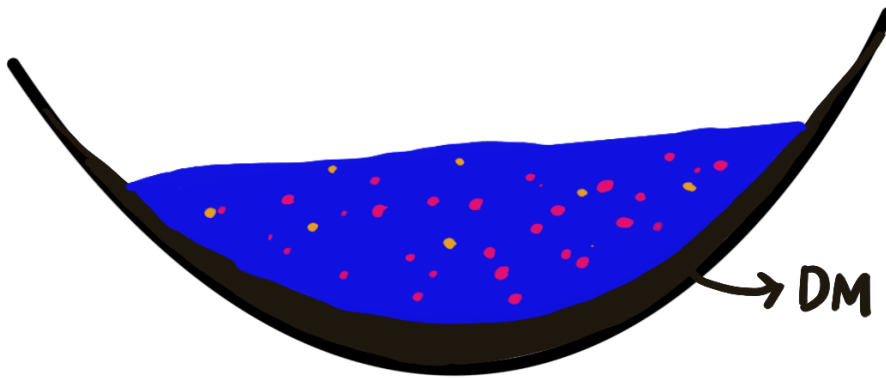
No



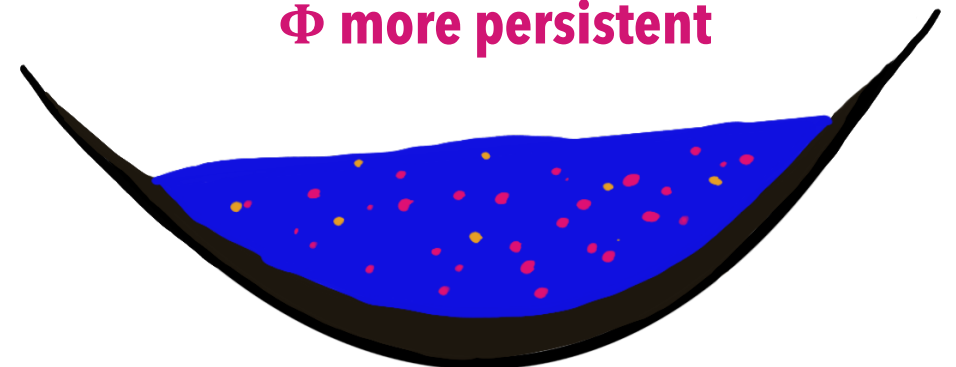
Radiation pressure prevents sustained collapse
 Φ decays



Yes

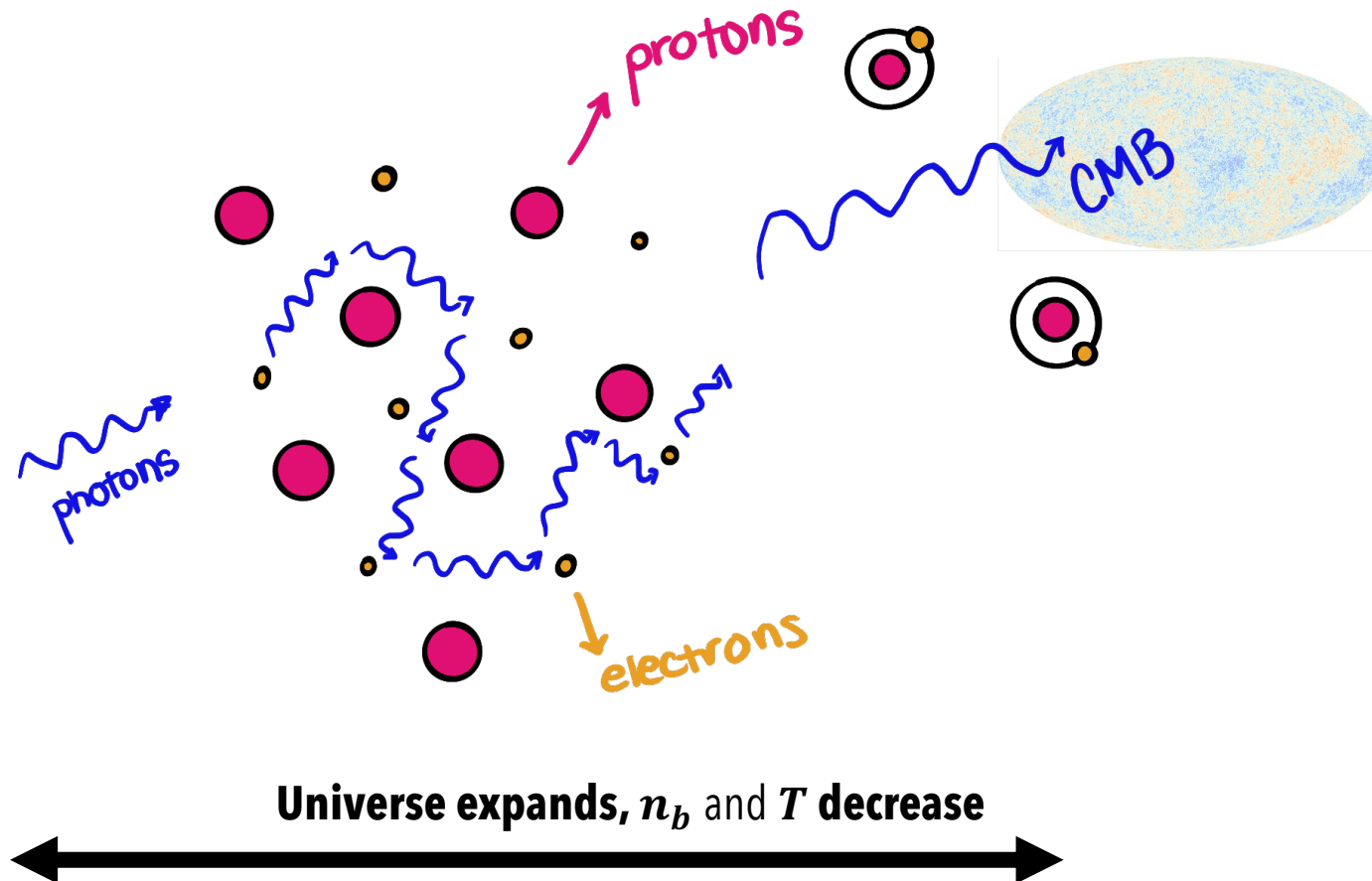


DM perturbations are pressureless
 Φ more persistent

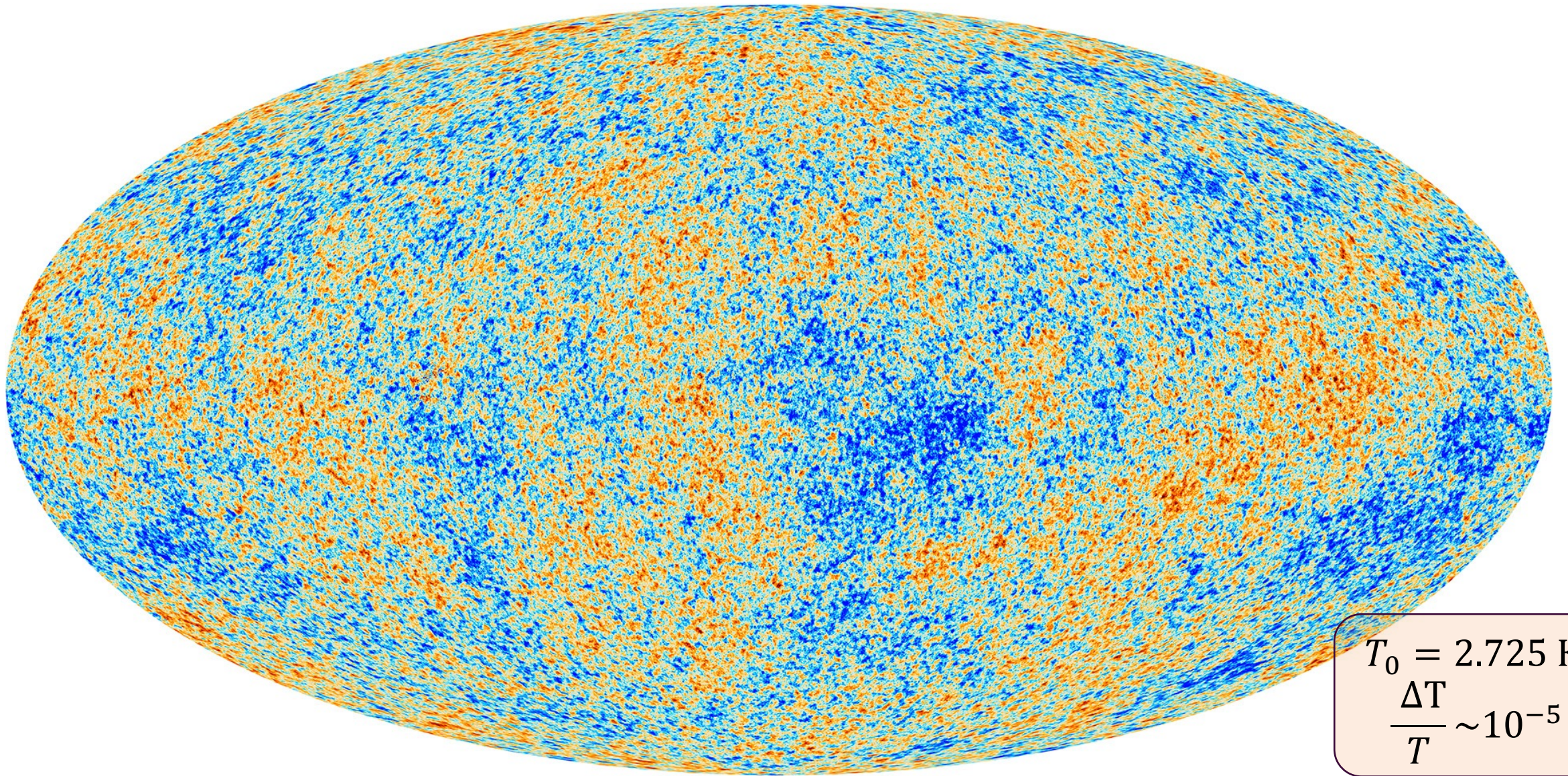


Cosmic Microwave Background (CMB)

In recombination, $t \simeq 380\,000$ years, $z \simeq 1100$, $T \simeq 0.3$ eV, the Universe becomes transparent to photons. **The CMB records the phase of each oscillation at the time of transparency.**

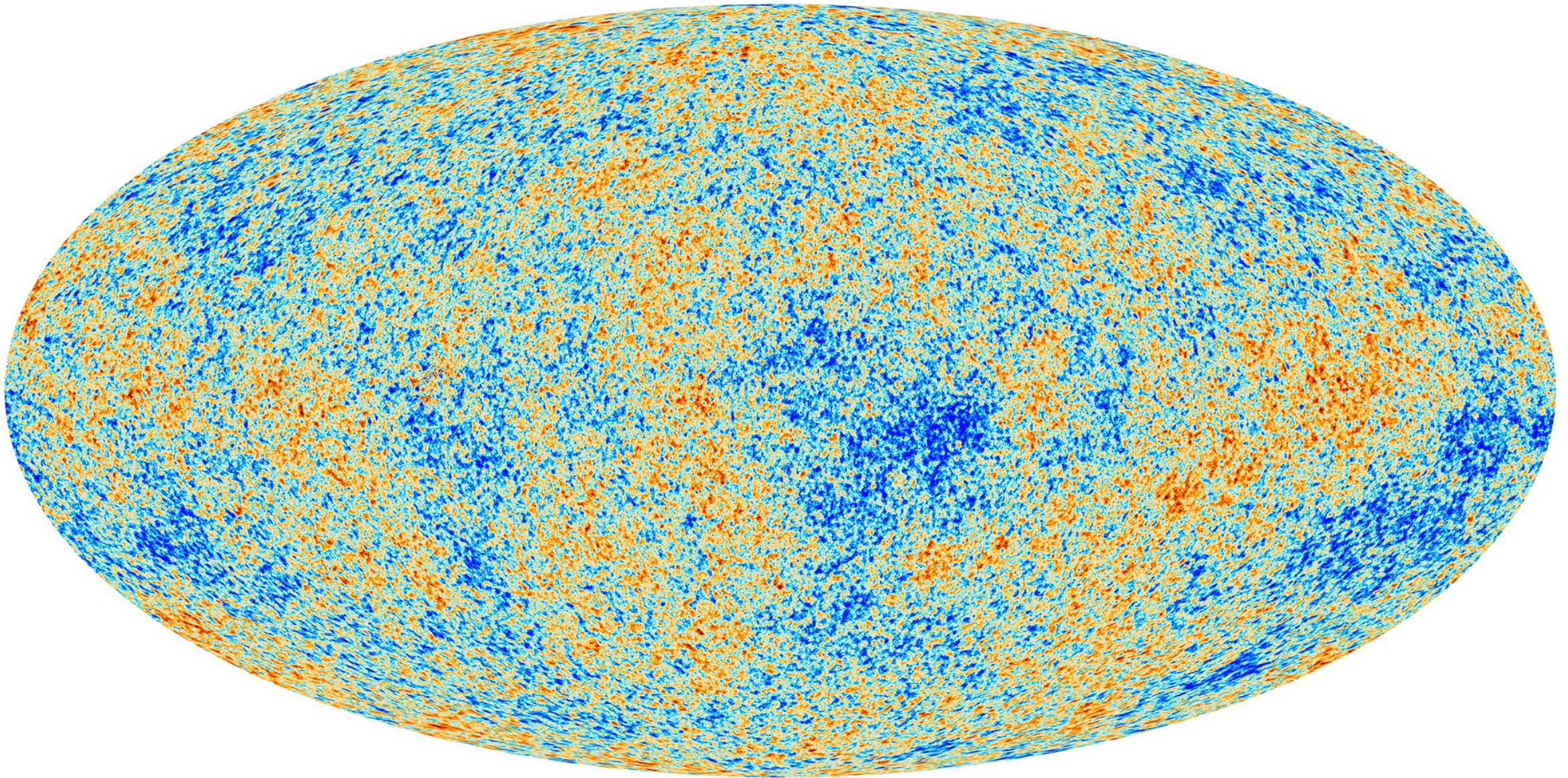


Cosmic Microwave Background (CMB)



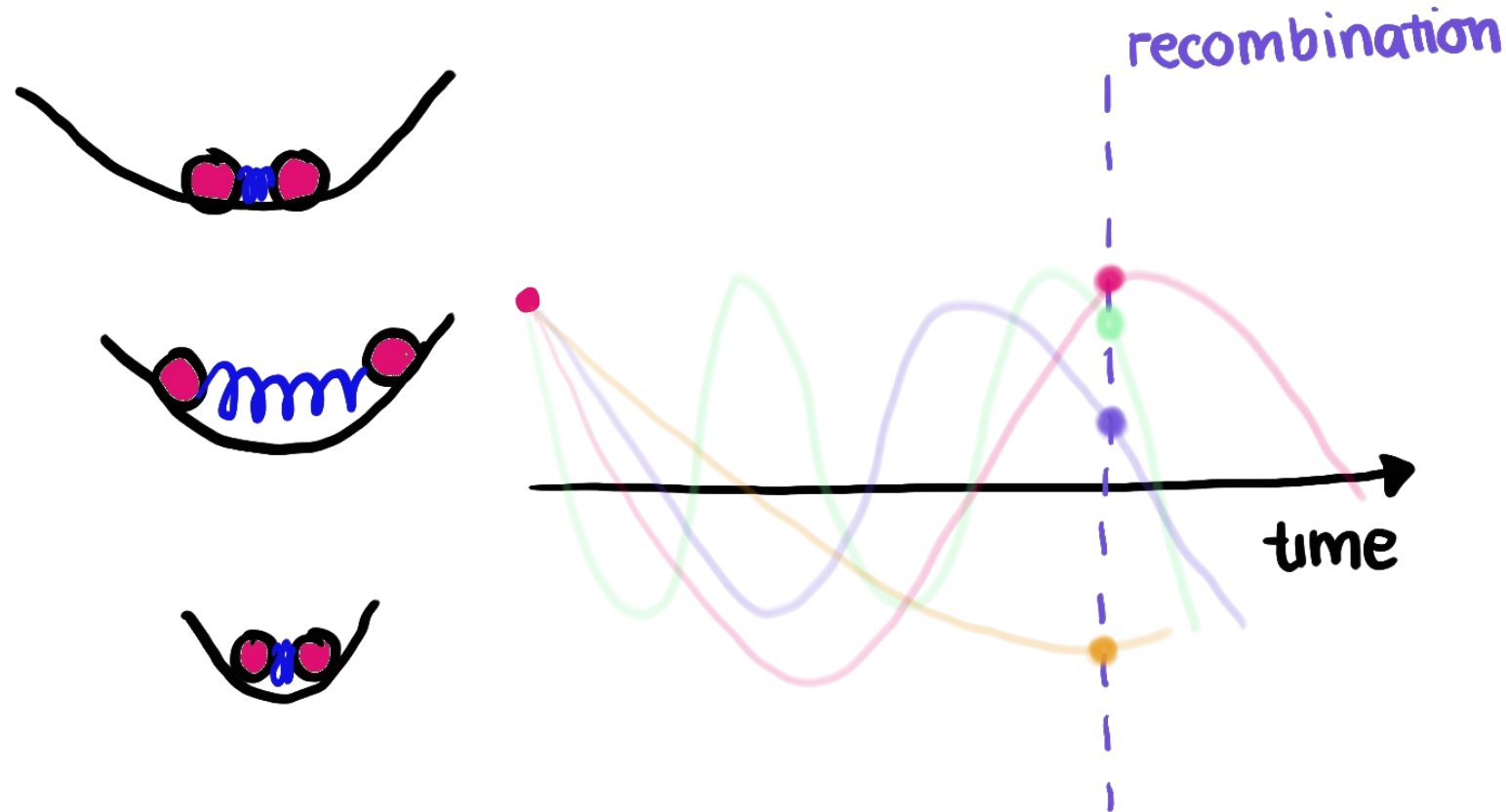
$$T_0 = 2.725 \text{ K}$$
$$\frac{\Delta T}{T} \sim 10^{-5}$$

How much temperature fluctuation is present on each angular scale?



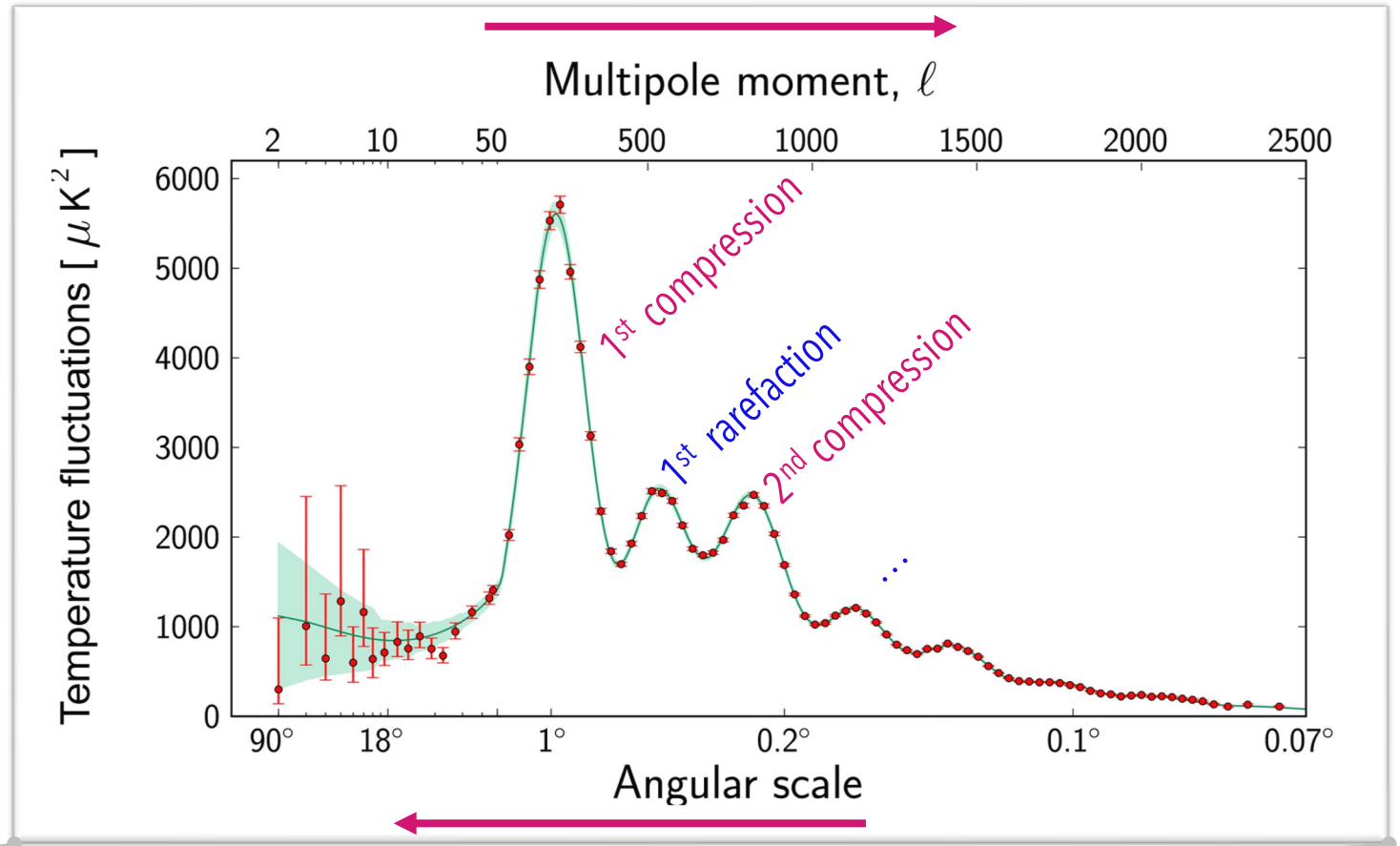
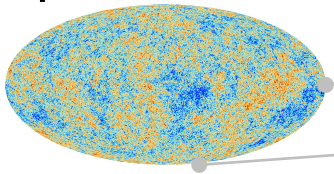
Cosmic Microwave Background (CMB)

In recombination, $t \simeq 380\,000$ years, $z \simeq 1100$, $T \simeq 0.3$ eV, the Universe becomes transparent to photons. **The CMB records the phase of each oscillation at the time of transparency.**



How much temperature fluctuation is present on each angular scale?

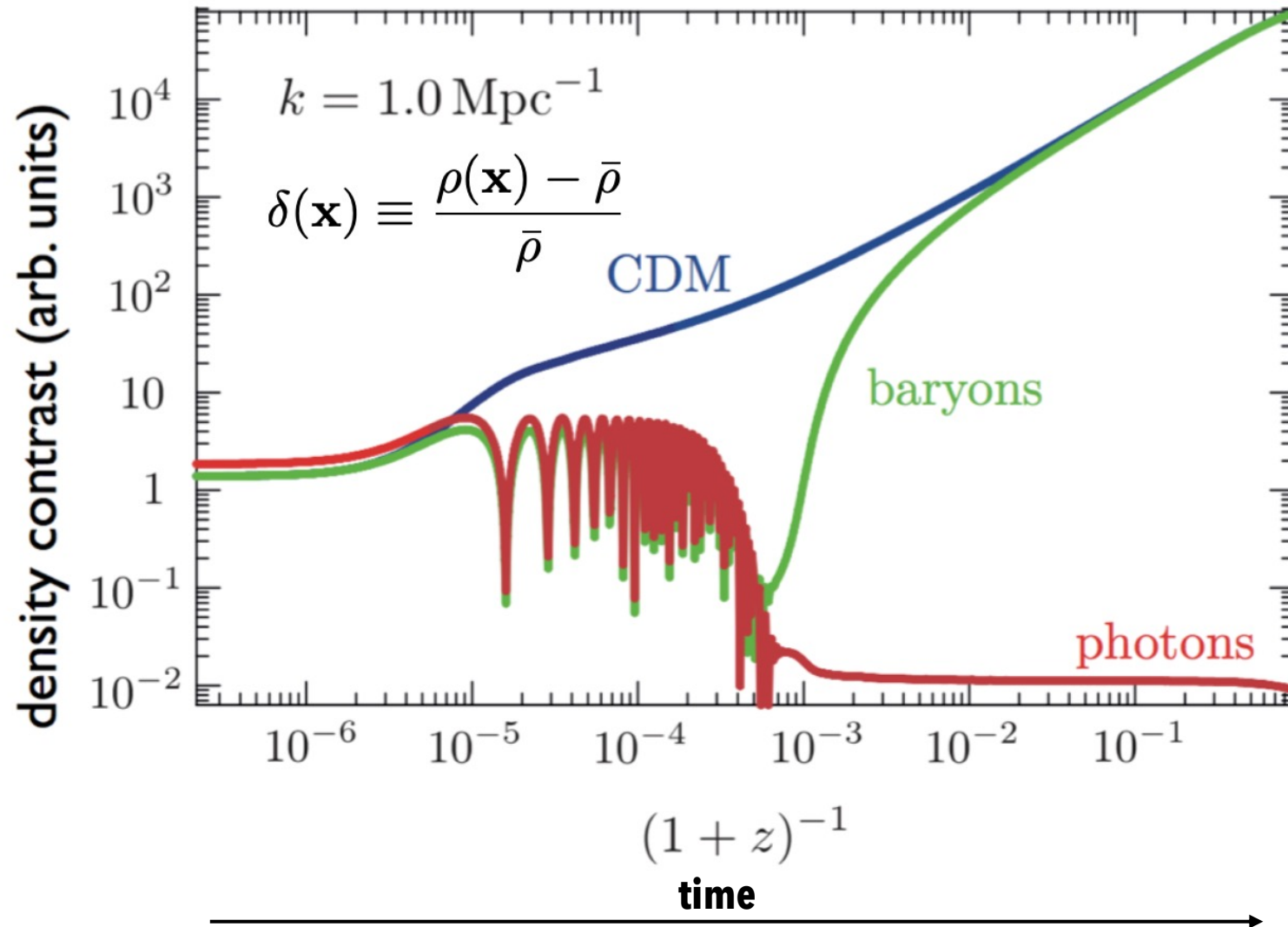
- Different perturbations had different angular sizes
- **Peak positions:** cosmic geometry
- **Odd-even modulation:** how much baryons is needed? (baryon loading)
- **Overall heights and higher-peak pattern:** gravitational potential evolution



$$\Omega_{\text{dm}} = 5.4 \Omega_b$$

Side note: It could have been anything (100x, 1000x, 10000x), but it's ~order of magnitude. Coincidence? Maybe not... 🙄

The growth of structure



Before recombination:

- baryons are tightly coupled to radiation, and photon pressure prevents their overdensities from growing significantly.
- Cold dark matter is pressureless and decoupled from radiation, allowing its perturbations to grow.

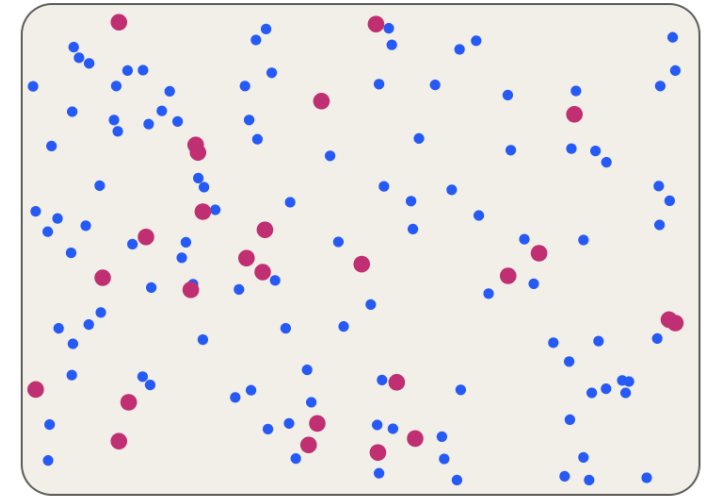
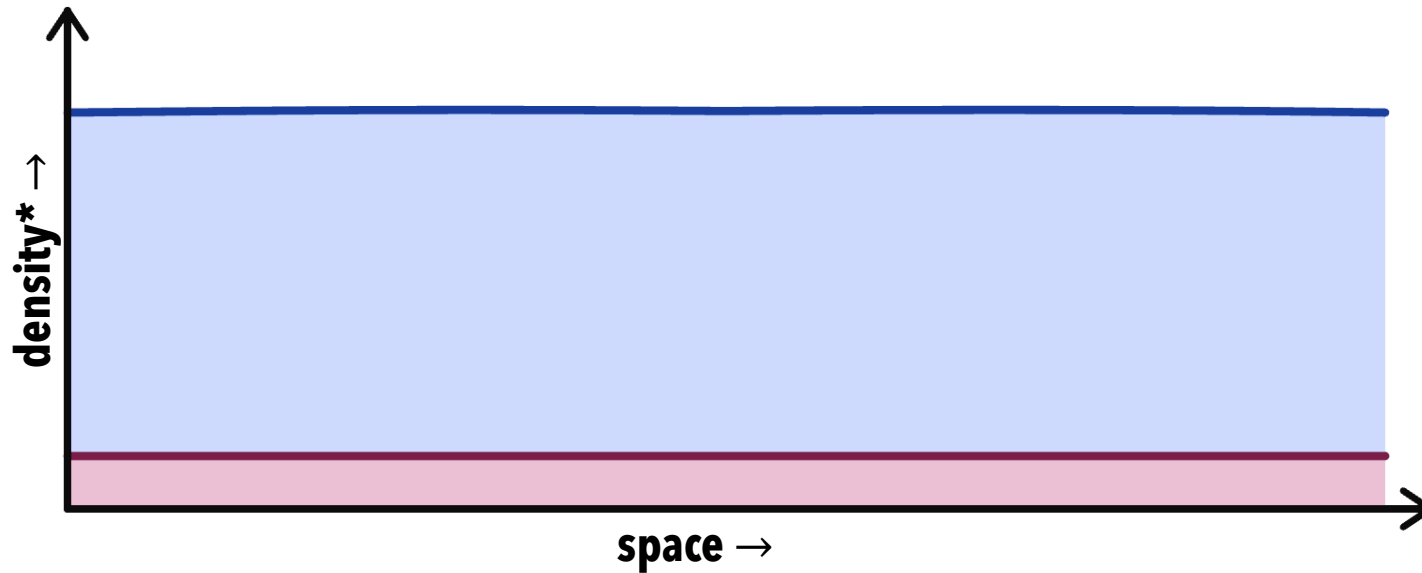
After recombination:

- baryons decouple from photons and rapidly fall into the gravitational potential wells already established by the dark matter.

The growth of structure

baryons

dark matter

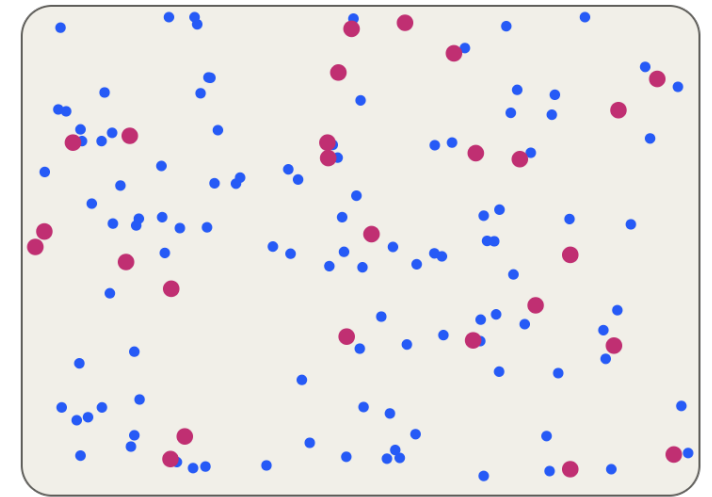
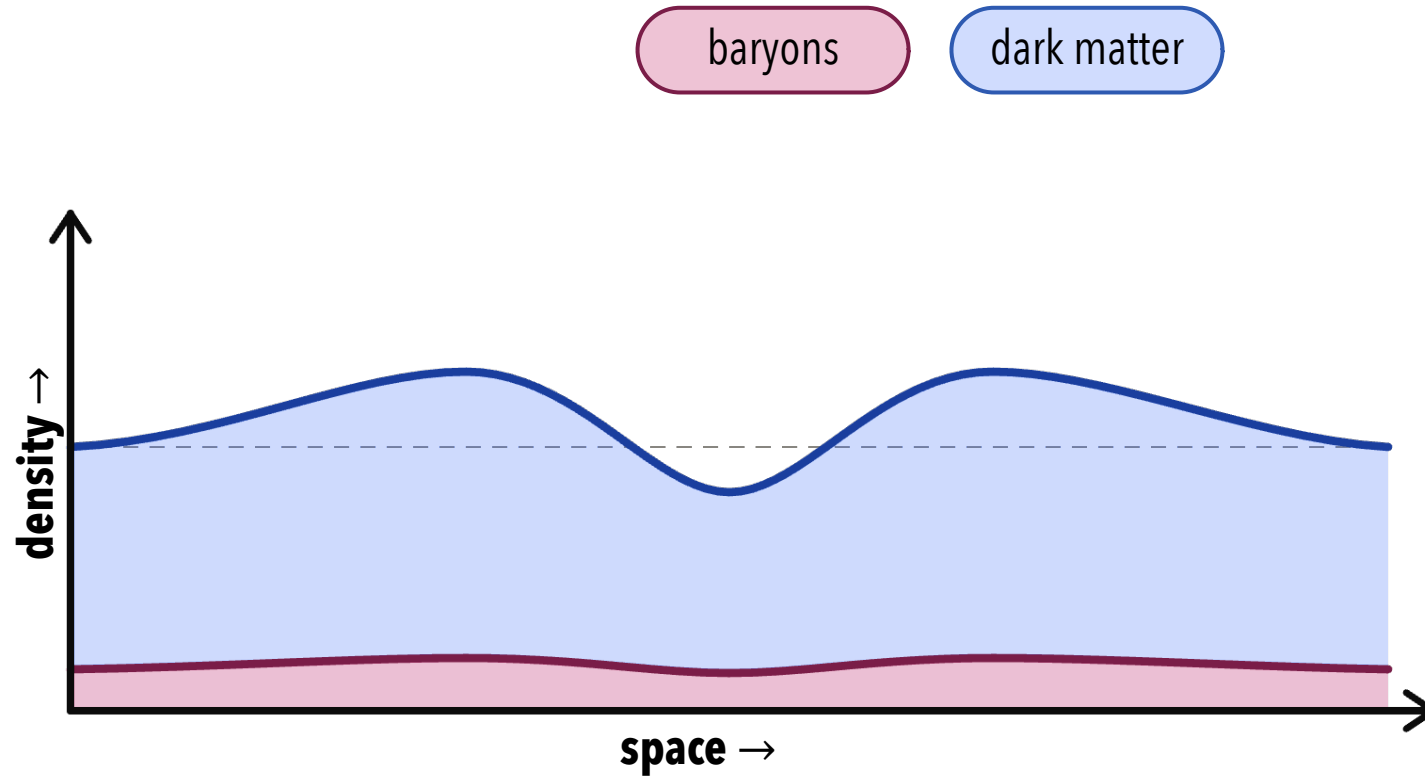


$t = 1$ second

both components nearly uniform

*the y-axis is not on a common scale across panels – compare shapes, not heights. x is comoving.

The growth of structure

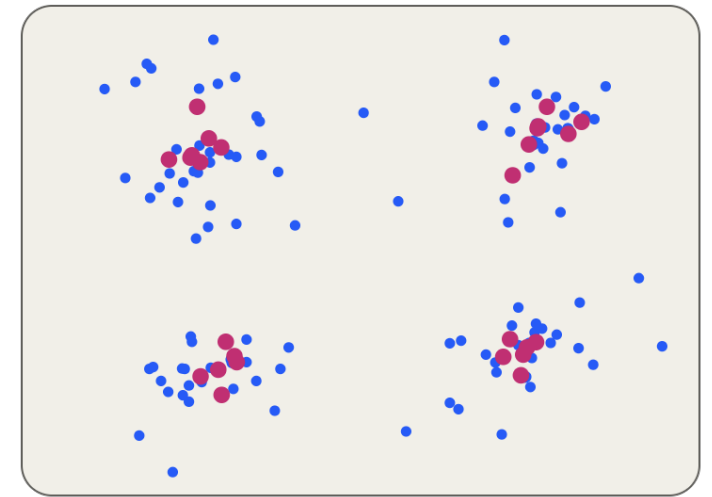
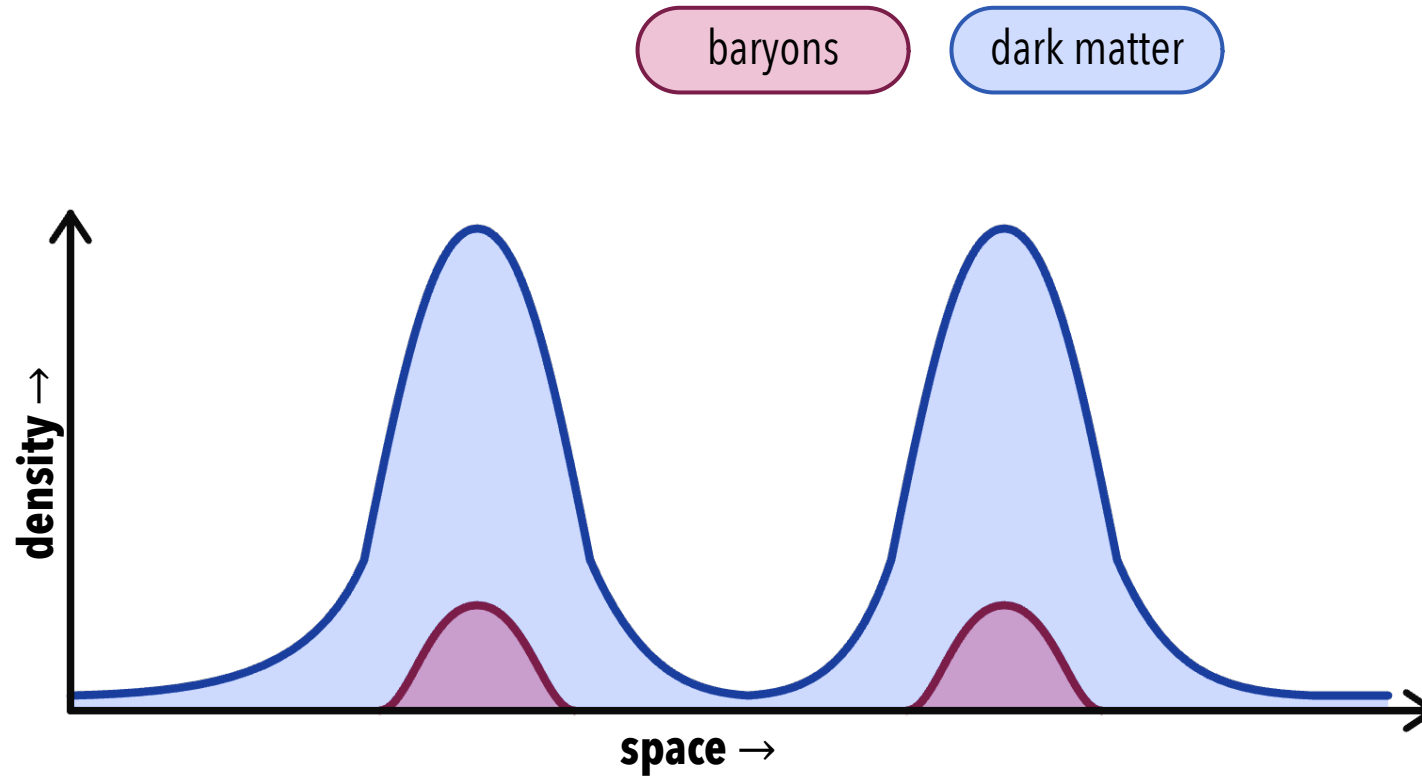


t = 1000 years

dark matter perturbations start to grow

*the y-axis is not on a common scale across panels – compare shapes, not heights. x is comoving.

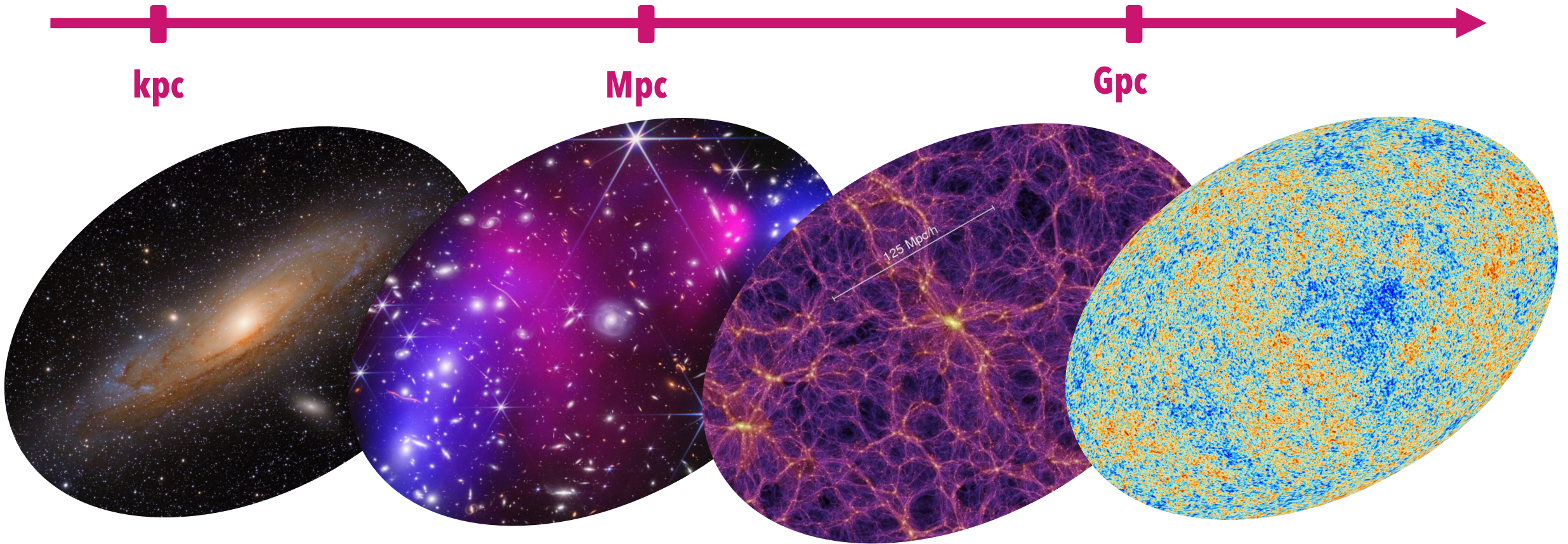
The growth of structure



$t = 10^8$ years
gas falls into the wells and forms galaxies

*the y-axis is not on a common scale across panels – compare shapes, not heights. x is comoving.

Take-home message: dark matter acting across the scales



A single non-baryonic dark-matter component consistently explains observations across these scales

Exercises

[Exercise 1.1.] In the MOND regime, Newton's second law is modified as

$$\mathbf{F} = m\mu\left(\frac{a}{a_0}\right)\mathbf{a},$$

where

$$\mu(x) \sim x \quad (x \ll 1).$$

Consider a test particle moving on a circular orbit of radius r around a baryonic mass M .

Show that, in the low-acceleration MOND regime $a \ll a_0$, the orbital velocity becomes independent of radius. Derive the asymptotic velocity in terms of G , M , and a_0 .

[Exercise 1.2.] In the early Universe, neutrons and protons are interacting through weak reactions such as

$$n + \nu_e \leftrightarrow p + e^-, \quad n + e^+ \leftrightarrow p + \bar{\nu}_e.$$

While these reactions are fast compared with the expansion rate, neutrons and protons remain in *chemical equilibrium*.

Use natural units,

$$k_B = c = 1,$$

and

$$\Delta m \equiv m_n - m_p = 1.293 \text{ MeV}, \quad T_w = 0.8 \text{ MeV}, \quad \tau_n = 880 \text{ s}.$$

Assume that efficient nucleosynthesis begins at $t_{\text{BBN}} \simeq 180 \text{ s}$.

[Exercise 1.2.]

(a) Assuming negligible chemical potentials and non-relativistic Maxwell-Boltzmann statistics, show that

$$\frac{n_n}{n_p} = e^{-\frac{\Delta m}{T}}.$$

Explain physically why the neutron abundance becomes smaller than the proton abundance as the Universe cools.

(b) Estimate n_n/n_p at $T_w = 0.8$ MeV.

The instantaneous freeze-out approximation is crude: weak freeze-out occurs gradually. For the remainder of the problem, take

$$\left(\frac{n_n}{n_p}\right)_w \sim \frac{1}{6}$$

as a representative neutron-to-proton ratio shortly after weak freeze-out.

[Exercise 1.2.]

(c) Before nucleosynthesis begins efficiently, free neutrons can decay,

$$n \rightarrow p + e^- + \bar{\nu}_e,$$

Assuming $t_w \ll t_{\text{BBN}}$, estimate the neutron-to-proton ratio when nucleosynthesis begins using

$$\left(\frac{n_n}{n_p}\right)_{\text{BBN}} \sim \left(\frac{n_n}{n_p}\right)_w e^{-\frac{t_{\text{BBN}}}{\tau_n}}$$

(d) Once the deuterium bottleneck opens, assume that essentially all surviving neutrons become bound into ${}^4\text{He}$. If $r \equiv n_n/n_p$, show that the primordial helium-4 mass fraction is approximately

$$Y_p \sim \frac{2r}{1+r}$$

Explain intuitively why the result is close to 25%, even though neutrons constitute only about one eighth of the nucleons when BBN begins.

[Exercise 1.2.]

(e) The characteristic rate for nuclear processing scales schematically as

$$\Gamma_{nuc} \sim n_b \langle \sigma v \rangle.$$

Consider two Universes at the same temperature but with different baryon densities n_b . Explain why

$$n_b \uparrow \Rightarrow D/H \downarrow$$

In your answer, explain the role of deuterium as the gateway to heavier nuclei and why a higher baryon density causes more deuterium to be processed into helium before nuclear reactions shut off.

[Exercise 1.2.]

(f) CMB gives a cold-dark-matter density of approximately $\omega_c = \Omega_c h^2 \sim 0.12$. Using information in the slides, find Ω_c / Ω_b .

Why does this comparison rule out the possibility that most cosmological dark matter consists simply of faint stars, brown dwarfs, planets, cold gas, or other ordinary non-luminous baryonic matter?